Add and Subtract Complex Numbers

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WHAT'S COVERED

In this lesson, you will learn how to add or subtract two complex numbers. Specifically, this lesson will cover:

1. Complex Numbers

A complex number is a number in the form $a + bi$, containing both a real and imaginary part. The imaginary part is followed by *i*, which is the imaginary unit, $\sqrt{-1}$.

Recall the following formulas for imaginary numbers:

FORMULA TO KNOW

Imaginary Number

$$i = \sqrt{-1}$$

$$i^2 = -1$$

We can combine complex numbers through addition and subtraction, just like we can add or subtract real numbers. The biggest distinction is that the real numbers and imaginary numbers remain separated, as is the case when combining like terms. This is the case with both addition and subtraction, where the only difference in the processes is the operation.

2. Adding Complex Numbers

We can think of a complex number addition problem as containing two addition sets. First, we'll add the real numbers together, which will constitute the first half of our solution. Secondly, we'll add the imaginary numbers together, and express that as the second half of the solution.

EXAMPLE Add $(5+6i)+(2+3i)$.

Start by lining up the two expressions. Add the real numbers together and then add the imaginary numbers together.

$$\begin{array}{r} (5+6i) \\ +(2+3i) \\ \hline \end{array}$$ Add real numbers, $5+2=7$

7 Add imaginary numbers, $6i+3i=9i$

$+9i$ Combine both parts

$7+9i$ Our solution

Sometimes, the addition of the two complex numbers has negative numbers in them. In these cases, we follow the same process when adding negative numbers: we can think of adding a negative number as subtracting a positive number.

EXAMPLE Add $(-3+4i)+(5-7i)$.

$$\begin{array}{r} (-3+4i) \\ +(5-7i) \\ \hline \end{array}$$ Add real numbers, $-3+5=2$

2 Add imaginary numbers, $4i+(-7i)=-3i$

$-3i$ Combine both parts

$2-3i$ Our solution

3. Subtracting Complex Numbers

When subtracting complex numbers, we again can break the problem down into two sets of subtraction: one set for all real numbers, and another set for imaginary numbers. The trickiest part with subtraction problems is paying attention to the sign of the numbers and the differences. This will be a particular concern when the subtraction problem contains negative numbers. Below are some examples of complex number subtraction:

EXAMPLE Subtract $(2+8i)-(-3+5i)$.

$$\begin{array}{r} (2+8i) \\ -(3+5i) \\ \hline \end{array}$$ Subtract real numbers, $2-3=-1$

-1 Subtract imaginary numbers, $8i-5i=3i$

$3i$ Combine both parts

$-1+3i$ Our solution

HINT

Be sure to still combine the real number and the imaginary number with addition. Although we are performing subtraction, remember that our general complex number is in the form $a+bi$. We would only see a minus sign between the two terms if the imaginary part was negative.

EXAMPLE Subtract $(8+3i)-(4-2i)$.

$$\begin{array}{rl} (8+3i) & \text{Subtract real numbers, } 8-4=4 \\ -(4-2i) & \\ \hline 4 & \text{Subtract imaginary numbers, } 3i-(-2i)=5i \\ +5i & \text{Combine both parts} \\ 4+5i & \text{Our solution} \end{array}$$

HINT

There is one thing, in particular, to note in the previous example. When subtracting the imaginary numbers, we subtracted a negative number, where we had 3*i* minus negative 2*i*. This can be thought of as adding a positive number, or 3*i* plus positive 2*i*.

SUMMARY

Complex numbers consist of a real part and an imaginary part. The square root of negative 1 is imaginary because no real number squared results in a negative number. When **adding or subtracting complex numbers**, you first combine the real numbers, and then combine the imaginary numbers.

FORMULAS TO KNOW

Imaginary Number

$i=\sqrt{-1}$

$i^2=-1$