Adding and Subtracting Complex Numbers

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WHAT'S COVERED

This tutorial covers how to add and subtract imaginary and complex numbers, through the exploration of:

1. Imaginary and Complex Numbers
2. Adding and Subtracting Imaginary Numbers
3. Adding and Subtracting Complex Numbers

1. Imaginary and Complex Numbers

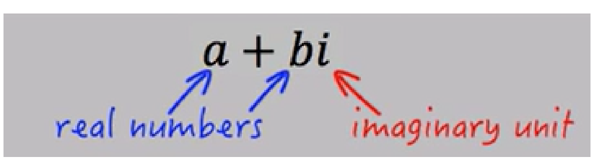
To review, the square root of a negative number is a non-real, or imaginary, number. The **imaginary unit *i*** is defined as the square root of -1.

FORMULA TO KNOW

Imaginary Number

$i = \sqrt{-1}$

A complex number is a value in the form below, in which *a* and *b* are real numbers, and *i* is the imaginary unit. In a **complex number**, *a* is the real part, and *b* times *i* is the imaginary part.



DID YOU KNOW

Complex numbers are used in fields such as engineering and physics.

TERM TO KNOW

Complex Number
A value of the form $a + bi$, where *a* and *b* are real numbers and *i* is the imaginary unit

2. Adding and Subtracting Imaginary Numbers

You may recall that the **product property for square roots** states that the square root of *a* times *b* is equal to the square root of *a* times the square root of *b*.

FORMULA TO KNOW

Product Property of Square Roots
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

You can apply the product property of square roots to solve equations involving the square root of a negative number, so that you are able to simplify your solution using imaginary numbers.

EXAMPLE You can rewrite the square root of -25 as follows, then apply the product property for square roots. Then you are able to simplify to arrive at your solution, which is an imaginary number.

$$\sqrt{-25} = \sqrt{25 \cdot -1} = \sqrt{25} \cdot \sqrt{-1} = 5i$$

The product property of square roots can also be applied when adding and subtracting imaginary numbers.

EXAMPLE Suppose you are solving the equation:

$$\sqrt{-4} + \sqrt{-49} - \sqrt{-9}$$

Applying the product property for square roots, you can simplify to:

$$2i + 7i - 3i$$

Now, *2i*, *7i*, and *3i* are all like terms. Therefore, you can combine them together by adding or subtracting their coefficients, to arrive at your final answer:

$$6i$$

3. Adding and Subtracting Complex Numbers

Adding and subtracting complex numbers is similar to combining like terms. You can add or subtract the real parts together, and add or subtract the coefficients of the imaginary parts together. You can add or subtract complex numbers in this way because of the commutative property of addition.

EXAMPLE Suppose you want to add the complex numbers:

$$(4+8i)+(2+3i)$$

You would start by adding your real parts, 4 and 2, together. Then you would add your imaginary parts, $8i$ and $3i$, together.

$$(4+2)+(8i+3i)$$

Combining your real parts together and your imaginary parts together gives you the final answer.

$$6+11i$$

TRY IT

Consider the following expression:

$$(11-6i)-(7+9i)$$

Combine like terms to subtract the complex numbers. +

Start by combining and subtracting your real parts, 11 minus 7, then combine and subtract your imaginary parts, $-6i$ minus $9i$, to arrive at your final answer.

$$(11-7)+(-6i-9i)=$$
$$4-15i$$

SUMMARY

Today you reviewed **imaginary numbers**, recalling that the square root of a negative number is non-real, or an imaginary number; the imaginary unit i is equal to the square root of -1. You also reviewed the definition of a **complex number**, which is a value in the form a plus bi, where a is the real part, and b times i is the imaginary part of the complex number. You learned how to apply the product property of square roots when **adding or subtracting imaginary numbers**. You also learned that when **adding or subtracting complex numbers**, you add or subtract the real parts and add or subtract the coefficients of the imaginary parts.

Source: This work is adapted from Sophia author Colleen Atakpu.

TERMS TO KNOW

Complex Number

A value of the form a + bi, where a and b are real numbers and 'i' is the imaginary unit.

FORMULAS TO KNOW

Imaginary Number

$i = \sqrt{-1}$

Product Property of Square Roots

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$