Adding and Subtracting Fractions

by Sophia

WHAT'S COVERED

This tutorial covers adding and subtracting fractions, through the exploration of:

1. Review of PEMDAS, the Order of Operations

To review, PEMDAS is an acronym used to remember the order of operations, which is the order in which you use operations when simplifying or solving problems, including problems involving fractions. PEMDAS stands for:

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

It's important to remember that multiplication and division are done together from left to right, and addition and subtraction are also done together from left to right.

2. Adding and Subtracting Fractions with Common Denominators

When you add or subtract fractions with the same, or common, denominators, you simply add or subtract the numerators.

EXAMPLE In the equation below, you are adding two fractions with common, or the same, denominators (the 5s). Therefore, you simply add the numerators of each fraction (2 and 1), which equals 3. The denominators stay the same.

$$\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$$

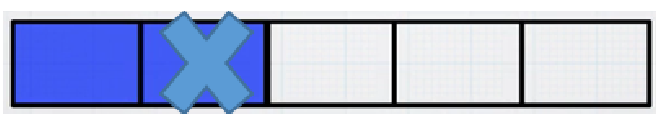
To think about this problem in a different way, look at the picture representation below. The bar is split into five pieces, which matches the 5 in the denominator. The pieces are all the same size because the denominators are all the same size. You start with 2/5, or 2 out of 5 pieces, shaded in, and then you add 1/5, or 1 piece, which equals a total of 3/5, or 3 out of 5 pieces, shaded in.

You can subtract the two fractions in the same way. When you subtract fractions, you subtract the numerators.

EXAMPLE Referring to our original equation, let's now subtract the two fractions. 2 minus 1 is 1. Again, the denominators stay the same.

$$\frac{2}{5}-\frac{1}{5}=\frac{1}{5}$$

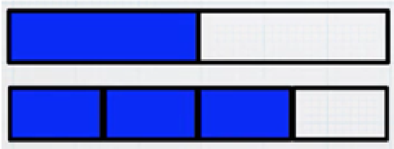
Looking at the bar representation, you start with 2 out of 5 pieces, or 2/5, shaded in, then subtract 1 piece (1/5), leaving 1 piece, or 1/5 of the bar, shaded in.



3. Adding and Subtracting Fractions with Uncommon Denominators

Suppose you want to add 1/2 and 3/4. Looking at the picture representation of these two fractions, you can see that there is a problem.

$$\frac{1}{2}+\frac{3}{4}=?$$



You can't simply combine the pieces together as you did in the last example, because the pieces are different sizes. However, if the denominators were the same, the pieces would be the same size, and you could add or subtract your numerators.

KEY CONCEPT

When adding or subtracting fractions with uncommon denominators, you need to convert them into equivalent fractions with common denominators.

EXAMPLE Let's evaluate $\frac{1}{2}+\frac{3}{4}$.

$\frac{1}{2}+\frac{3}{4}$	Our Expression
$\frac{1\times4}{2\times4}+\frac{3}{4}=\frac{?}{?}$	Start by multiplying the denominators. In our first fraction, we multiply the denominator, 2, by the denominator of our second fraction, 4. This equals 8. (Remember, you must multiply the numerator by 4 as well, to keep the fraction's value the same. Multiplying the numerator and denominator by the same number is like multiplying the fraction by 1, which does not change the value of the fraction)
$\frac{4}{8}+\frac{3\times2}{4\times2}=\frac{?}{8}$	Next, multiply the denominator of the second fraction, 4 by the denominator of the first fraction, 2. This makes the common denominator 8. We must also multiply the numerator of the second fraction by 2 to ensure the value of the fraction stays the same.
$\frac{4}{8}+\frac{6}{8}=\frac{10}{8}$	Now that our denominators are both 8, we can add the numerators together, and leave the denominator the same. This gives us 10/8.
$\frac{10}{8}=\frac{5}{4}$	To simplify, or reduce, the fraction, we can divide both the numerator and denominator by 2, which equals 5/4.
$\frac{5}{4}$	Our Solution

HINT

This method of finding a common denominator will always work, but it may not give you the least or smallest common denominator, and some simplification might be necessary at the end.

4. Finding Least Common Denominators of Fractions

Finding the least common denominator between fractions can eliminate some common factors, making simplification easier. To find the least common denominator, you find the smallest number that is a multiple of the denominators of each fraction.

Consider the last example again to see how you can find the least common denominator.

EXAMPLE Let's keep our original addition problem:

$\frac{1}{2}+\frac{3}{4}$	Our Expression
$\frac{1}{2}+\frac{3}{4}=\frac{?}{?}$	The multiples of our first denominator (2) are 2, **4**, 6, 8, 10, 12, and so on. The multiples of our second denominator (4) are **4**, 8, 12, 16, 20, 24, and so on.
$\frac{1\times2}{2\times2}+\frac{3}{4}=\frac{?}{4}$	The smallest common multiple is 4, so we can rewrite our fractions using 4 as our common denominator. To convert the denominator of 1/2 to 4, you multiply by 2 in the denominator and numerator. The second fraction can stay the same, as it already has a denominator of 4.
$\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$	Finally, add together your numerators. 2 and 3 equals 5.
$\frac{5}{4}$	Our Solution

5. Using the Order of Operations with Fractions

You can also use order of operations, or PEMDAS, with fractions. Suppose you are solving the following problem.

EXAMPLE Evaluate $\frac{3}{4}+2\left(\frac{1}{3}\right)-\frac{1}{2}$.

$\frac{3}{4}+2\left(\frac{1}{3}\right)-\frac{1}{2}$	Our Expression
$\frac{3}{4}+\underbrace{\left(\frac{2}{1}\right)\left(\frac{1}{3}\right)}-\frac{1}{2}$	Start with multiplication and multiply 2 times 1/3. The whole number 2 can be written as a fraction with the denominator of 1.
$\frac{3}{4}+\frac{2}{3}-\frac{1}{2}$	Evaluate 2/1 times 1/3. This equals 2/3.

$$\frac{(3\times3)}{(4\times3)}+\frac{(2\times4)}{(3\times4)}-\frac{(1\times6)}{(2\times6)}=\frac{?}{12}$$

Next, we can proceed to addition/subtraction from left to right. We need to find a common denominator, and looking at our denominators 4, 3, and 2, our least common denominator is 12. Now, multiply our current numerators and denominators by the number that will make each denominator 12.

$$\frac{9}{12}+\frac{8}{12}-\frac{6}{12}=\frac{11}{12}$$

Now that the denominators are all the same, we can add/subtract the numerators. 9 plus 8 minus 6 equals 11.

$$\frac{11}{12}$$

Our Solution

SUMMARY

Today you learned about **adding and subtracting fractions**, and that to do so, fractions must have the same or **common denominator**. You also learned that to find a common denominator, you can always multiply the denominators together, but you may need to simplify your answer. Lastly, you learned that **the order of operations (PEMDAS)** must be used when simplifying all expressions, including expressions with fractions.

Source: This work is adapted from Sophia author Colleen Atakpu.