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Adding and Subtracting Polynomials

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WHAT'S COVERED

In this lesson, you will learn how to add or subtract two polynomial expressions. Specifically, this lesson will cover:

1. Combining Like Terms

When adding and subtracting polynomials, it really is an exercise in combining like terms. Like terms have the same variables, which are raised to the same power.

EXAMPLE $2x^2$ and $7x^2$ are like terms, because they both contain *x* raised to the second power.

EXAMPLE $5x^3$ and $5x^2$ are NOT like terms. Although they both start with 5 and also contain the variable *x*, their exponents are different: one is *x* cubed, and the other is *x* squared.

BIG IDEA

Like terms can be combined by adding their coefficients. For instance, combining $2x^2$ and $7x^2$ gives us $9x^2$.

If coefficients are negative, we can think of it as subtraction: combining $2x^2$ and $-7x^2$ gives us $-5x^2$.

2. Identifying Like Terms in Polynomials

Polynomials have several terms, and likely contain several different types of terms. When we add and subtract polynomials, it is important to identify like terms and organize the terms by their type, so that we can easily add or subtract their coefficients.

EXAMPLE Rewrite the following polynomial by combining like terms:

$$2xy+4x^2+3x^2y-x^2$$

Let's first examine the variables and exponents to determine which terms are like terms and can be combined. When looking at the term $2xy$, we scan the other terms to see if we have other terms with an x and a y. We see $3x^2y$, but is this a like term to $2xy$? It actually isn't. This is because $3x^2y$ has two factors of x (the exponent of 2 is attached to the x), whereas $2xy$ only has 1 factor of x.

We do however have the like terms $-x^2$ and $4x^2$. These two terms can be combined as $3x^2$ because -1 and 4 add to 3.

Let's rewrite this polynomial with the combined like terms:

$$3x^2y+3x^2+2xy$$

HINT

It is standard to write the terms in order of their degree, from highest to lowest. In the example above, $3x^2y$ is written first because its degree is 3 (since the x variable had an exponent of 2 and there is an implied exponent of 1 for y), whereas the other terms have a degree of 2.

3. Adding Polynomials

When adding polynomials, it helps to write the problem vertically, so that you can align the like-terms, making the addition of the coefficients easier to do in your head. We'll see in the section below that this is the same process for subtraction.

EXAMPLE Add the following two polynomials:

$$(3x^3+2x-5)+(2x^3+5x^2-7x)$$

Let's first set this up by writing the problem vertically:

$$\begin{array}{r} (3x^3+2x-5) \\ \underline{+(2x^3+5x^2-7x)} \end{array}$$

Before we complete the addition, let's take a closer look at the vertical set up. The whole point of writing the addition vertically is to line up the like terms. But we noticed that there were some terms in one polynomial that were not in the other, so like terms are not lining up together. In this case, it helps to write a term with a coefficient of zero, as a placeholder to keep everything vertically aligned.

$$\begin{array}{r} (3x^3+0x^2+2x-5) \\ \underline{+(2x^3+5x^2-7x+0)} \end{array}$$

Now we can add the two polynomials together, combining the coefficients of like terms:

$$
\begin{array}{r}
(3x^3+0x^2+2x-5) \\
+(2x^3+5x^2-7x+0) \\
\hline
5x^3+5x^2-5x-5
\end{array}
$$

4. Subtracting Polynomials

When subtracting polynomials, we follow the same procedure, only we subtract coefficients rather than add them. The only tricky thing to watch out for is subtracting a negative number. This should be thought of as adding a positive number.

EXAMPLE Subtract the following two polynomials:

$$(6x^3-4)-(2x^2-7x-3)$$

Once again, we'll write the problem vertically and add terms with coefficients of 0 to keep everything aligned. It also helps to group each polynomial in parentheses, so that we can get the subtraction out in front and still be aware of the signs of each term within the polynomial.

$$
\begin{array}{r}
(6x^3+0x^2+0x-4) \\
-\underline{(0x^3+2x^2-7x-3)}
\end{array}
$$

Now we can subtract the two polynomials, evaluating for the coefficients of like terms:

$$
\begin{array}{r}
(6x^3+0x^2+0x-4) \\
-(0x^3+2x^2-7x-3) \\
\hline
6x^3-2x^2+7x-1
\end{array}
$$

HINT

Was any part of that subtraction tricky? It was probably easy enough to get the coefficients of the first two terms:

$$6x^3-0x^3=6x^3$$
$$0x^2-2x^2=-2x^2$$

However, sometimes it can be difficult to remember signs with subtraction, especially for the last two coefficients:

$$0x-(-7x)=0x+7x=7x$$
$$-4-(-3)=-4+3=-1$$

SUMMARY

When **combining like terms**, add the coefficients and keep the variables and powers the same.

Identifying like terms in polynomials is where the terms have the same variables and associated variable power. When **adding polynomials**, a term with an addition sign or no sign indicates that the coefficient for the term is positive. When **subtracting polynomials**, a term with a subtraction sign before it signifies that the coefficient for the term is negative.

