Adding and Subtracting Rational Expressions

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WHAT'S COVERED

In this lesson, you will learn how to add or subtract two rational expressions with an uncommon denominator. Specifically, this lesson will cover:

1. Adding and Subtracting Numeric Fractions

Reviewing how to add and subtract numeric fractions is helpful when learning how to add and subtract algebraic fractions. This is because the same general principle is applied in both cases. In order to add two fractions, we want to express each fraction such that the denominators are the same. Then we can simply add or subtract across the numerators and retain the common denominator.

EXAMPLE When adding $\frac{5}{7}+\frac{3}{7}$, this is fairly straightforward. The denominators are already equivalent, so we can just add the numerators 5 and 3 to get a sum of $\frac{8}{7}$.

When the denominators are not the same, we need to write equivalent equations with the same denominator.

EXAMPLE Consider the subtraction problem $\frac{5}{6}-\frac{1}{2}$.

$\frac{5}{6}-\frac{1}{2}$	Multiply $\frac{1}{2}$ by $\frac{3}{3}$
$\frac{5}{6}+\frac{1}{2}\cdot\frac{3}{3}$	Evaluate multiplication
$\frac{5}{6}-\frac{3}{6}$	Subtract numerators
$\frac{5-3}{6}$	Evaluate numerator
$\frac{2}{6}$	Simplify
$\frac{1}{3}$	Our solution

2. Adding Rational Expression with Common Denominators

When the denominators between two factions are the same, adding and subtracting is much easier. This is also the case with rational expressions. Below is an example of adding rational expressions with the same denominator:

↪ EXAMPLE Add $\frac{2x+2}{4x-4}+\frac{3x-6}{4x-4}$.

Since the denominators are the same, we can simply add the numerators together and keep the denominator.

$\frac{2x+2}{4x-4}+\frac{3x-6}{4x-4}$	Add numerators, keep denominator
$\frac{(2x+2)+(3x-6)}{4x-4}$	Evaluate numerator
$\frac{5x-4}{4x-4}$	Our solution

☆ BIG IDEA

Adding and subtracting rational expressions with common denominators operates in the same way as adding and subtracting numeric fractions with common denominators. We add or subtract across the numerators, and keep the denominator the same.

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3. Adding and Subtracting Rational Expressions with Uncommon Denominators

When adding or subtracting rational expressions without a common denominator, we must re-express the fractions so that they do have a common denominator. That way, we can add or subtract as we did in the example above.

The easiest way to find a common denominator between two algebraic fractions is to follow these steps:

1. Multiply both the numerator and the denominator of the first fraction by the denominator of the second fraction.
2. Repeat the process with the second fraction: multiply both the numerator and the denominator of the second fraction by the denominator of the first fraction.
3. Add or subtract numerators and keep the denominator the same.

↪ EXAMPLE Subtact $\frac{x+2}{x}-\frac{7}{x-3}$.

Our first task is to find a common denominator. To do this, multiply the numerator and the denominator of the first fraction $\frac{x+2}{x}$ by a fraction with the denominator of the second fraction, $x-3$, in both the numerator and denominator, $\frac{x-3}{x-3}$.

$\frac{x+2}{x}$	Multiply by a fraction made from the denominator of the second fraction, $\frac{x-3}{x-3}$
$\frac{x+2}{x}\left(\frac{x-3}{x-3}\right)$	Multiply numerators together by FOILing; denominator is product of the two denominators
$\frac{x^2-3x+2x-6}{x(x-3)}$	Combine like terms
$\frac{x^2-x-6}{x(x-3)}$	Our new first fraction

We will repeat this process with the second fraction. Multiply the numerator and the denominator of the second fraction $\frac{7}{x-3}$ by a fraction with the denominator of the first fraction, x, in both the numerator and denominator, $\frac{x}{x}$.

$\frac{7}{x-3}$	Multiply by a fraction made from the denominator of the first fraction, $\frac{x}{x}$
$\frac{7}{x-3}\left(\frac{x}{x}\right)$	Multiply numerators together; denominator is product of the two denominators
$\frac{7x}{x(x-3)}$	Our new second fraction

Now that both fractions have the same denominator, we can subtract.

$\frac{x+2}{x}-\frac{7}{x-3}$	Replace fractions with results from above
$\frac{x^2-x-6}{x(x-3)}-\frac{7x}{x(x-3)}$	Subtract numerators; denominator stays the same
$\frac{(x^2-x-6)-(7x)}{x(x-3)}$	Evaluate numerator
$\frac{x^2-x-6-7x}{x(x-3)}$	Combine like terms
$\frac{x^2-8x-6}{x(x-3)}$	Our solution

The expression $\frac{x+2}{x}-\frac{7}{x-3}$ can be written as $\frac{x^2-8x-6}{x(x-3)}$.

HINT

This can be the easier strategy, however, this can get messy, because there may be common factors in the numerator and denominator that should be canceled, but not necessarily easy to spot.

EXAMPLE Add $\frac{x+1}{5x}+\frac{2x+3}{3x^2}$.

$\frac{x+1}{5x}+\frac{2x+3}{3x^2}$	Multiply each fraction by a fraction made up with the denominator of the other
$\frac{x+1}{5x}\left(\frac{3x^2}{3x^2}\right)+\frac{2x+3}{3x^2}\left(\frac{5x}{5x}\right)$	Evaluate the multiplication in each numerator
$\frac{3x^3+3x^2}{5x(3x^2)}+\frac{10x^2+15x}{5x(3x^2)}$	Add numerators; denominator stays the same
$\frac{(3x^3+3x^2)+(10x^2+15x)}{5x(3x^2)}$	Combine like terms
$\frac{3x^3+13x^2+15x}{5x(3x^2)}$	Simplify numerator by factoring out common term, x
$\frac{x(3x^2+13x+15)}{5x(3x^2)}$	Cancel x from both numerator and denominator
$\frac{3x^2+13x+15}{5(3x^2)}$	Our solution

The expression $\frac{x+1}{5x}+\frac{2x+3}{3x^2}$ can be written as $\frac{3x^2+13x+15}{5(3x^2)}$.

SUMMARY

Recall when **adding and subtracting numeric fractions**, the fractions must have a common denominator. To **add or subtract rational expressions**, write each fraction with a common denominator then add or subtract the numerators, keeping the denominator the same. To find a common denominator, multiply both the numerator and the denominator of the first fraction by the denominator of the second fraction. Repeat this process with the second fraction.