"And" Probability for Dependent Events

by Sophia

WHAT'S COVERED

This tutorial will cover the general "and" probability formula for dependent events. Our discussion breaks down as follows:

1. “And” Probability for Dependent Events

In this tutorial, you're going to learn about the general **"and" probability formula for dependent events**. In a separate tutorial, we talked about "and" probability for independent events, which had a special multiplication rule where you simply multiplied the probability of each event together. This, on the other hand, has a general multiplication rule.

EXAMPLE Suppose there's a famous television show called "Go For Broke.” On the show, a person gets to roll a die to determine which jar they're going to draw a colored chip from. If the chip is green, they will win a prize, and if the chip is red, they won’t win anything.

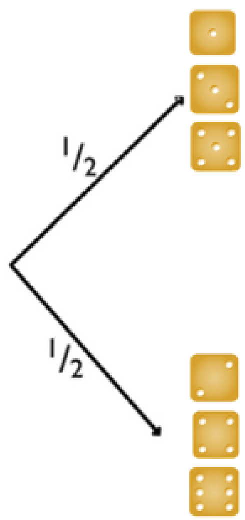
- If a person rolls an even number, they select from the A jar that contains seven green chips and 13 red chips. Choosing from this jar gives you a decent probability to win the prize.
- If a person rolls an odd number, they select from the B jar that contains five green chips and 25 red chips. This jar doesn’t give you a very good likelihood of winning.

Roll an Even:
Pick from Jar A

Roll an Odd:
Pick from Jar B

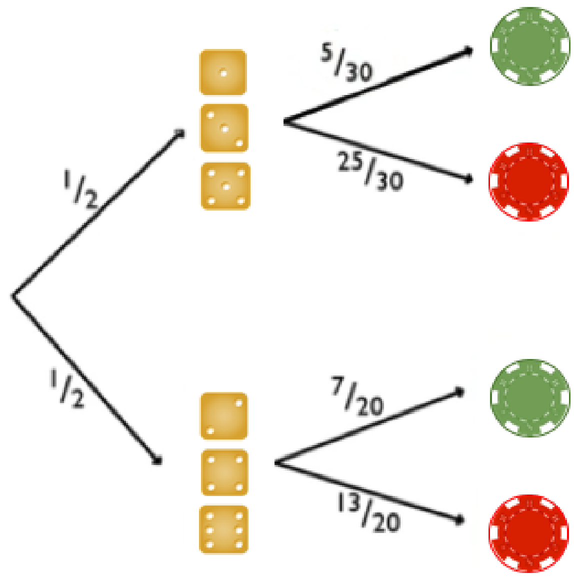
The probability of picking a red chip depends on the die roll. That means that these two events are dependent. The probability of picking a red chip increases if you roll an odd number versus if you roll an even number.

Let's look at it in a tree diagram:



There's a 1/2 probability of rolling an odd and a 1/2 probability of rolling an even. However, that's where the similarities end:

- If you roll the odd, there's a 5/30 probability of getting a green chip and a 25/30 probability of getting a red chip.
- If you roll an even, there's a 7/20 probability of getting a green chip and a 13/20 probability of getting a red.



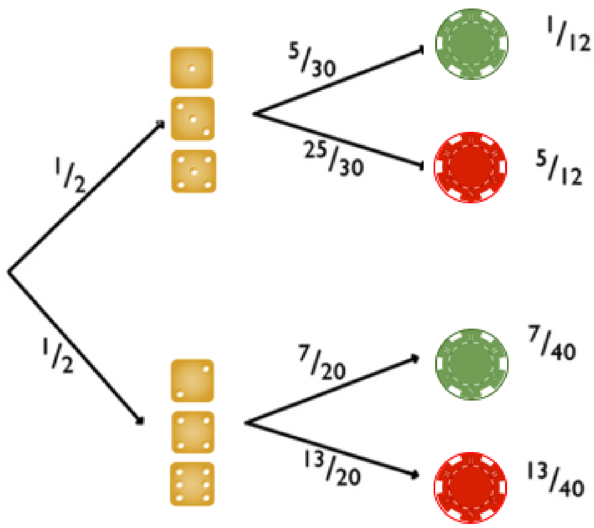
Using the tree diagram, we can find the conditional probability of selecting a green, given you have already rolled an odd number:

$$P(Green \mid Odd) = \frac{5}{30}$$

But what if we want to know the probability of rolling an odd AND getting a green chip? For this probability, we would multiply the probability of an odd, P(Odd), times the probability of green, given odd, P(Green | Odd).

This leads us to the general probability formula, the "and" probability formula for dependent events. It's the probability of the first event times the probability that the second event occurs, given the first event occurred.

When we multiply it all out, you end up with the following tree diagram:



FORMULA TO KNOW

"And" Probability for Dependent Events

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

TERM TO KNOW

"And" Probability for Dependent Events

The probability that two events both occur is the probability of the first event times the conditional probability that the second event occurs, given that the first already has. Also known as the "General Multiplication Rule".

2. Independence Revisited

Recall that when two events are independent, this really means that knowing the outcome of the first event, A, doesn't change the probability of the second event (B).

So, for independent events, we can say the probability that B occurs given that we know that A happened is the same as just the probability of B--whether or not we knew A was happening. In other words, A has no effect on the probability of B.

For Independent Events:
$P(B \mid A) = P(B)$

What does that mean within the "and" probability formula? That means the probability of A and B is equal to the probability of A times the probability of B given A, which we know this by the general multiplication rule.

But for independent events, probability of B given A is the same as the probability of B. This formula looks familiar. That's the special multiplication rule for independent events. The special multiplication rule is a special version of the general multiplication rule.

For Independent Events:
$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$
$P(A \text{ and } B) = P(A) \cdot P(B)$

BIG IDEA

The special multiplication rule for independent events is a special version of the general multiplication rule.

SUMMARY

The joint probability of two events occurring together, either concurrently or consecutively, is equal to the probability of the first event times the probability of the second event given that the first event occurred. It's easiest to think about this like you're going down the branches of a tree diagram. The general multiplication rule works for both independent and dependent events.

Good luck!

TERMS TO KNOW

And Probability for Dependent Events

The probability that two events both occur is the probability of the first event times the conditional probability that the second event occurs, given that the first already has. Also known as the "General Multiplication Rule)".

FORMULAS TO KNOW

And Probability for Dependent Events

$P(A \text{ and } B) = P(A) \cdot P(B|A)$