Applying Properties of Logarithms

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WHAT'S COVERED

In this lesson, you will learn how to evaluate a logarithmic expression using the properties of logarithms. Specifically, this lesson will cover:

1. Applying the Log-Exponent Relationship

Keeping the log-exponent relationship in mind can help us rewrite logarithmic expressions and evaluate them. Exponents and logarithms are inverse operations, and we can rewrite these two kinds of expressions in the following way:

- Exponential expression: $y = b^x$
- Logarithmic expression: $\log_b(y) = x$

We can connect these two expressions in the following way:

FORMULA TO KNOW

Logarithmic Form to Exponential Form
$\log_b(y) = x \leftrightarrow y = b^x$

EXAMPLE Evaluate $\log_3(9) + \log_3(27) - \log_3(3)$.

We can apply various properties of logs to simplify and evaluate this expression. First, let's use the inverse relationship between exponents and logs to think about how to evaluate each logarithmic term individually:

$\log_3(9) = x$ Rewrite as an exponential expression

$3^x = 9$	$x = 2$ makes this true
$3^2 = 9$	Use the Log-Exponent Relationship
$\log_3(9) = 2$	Our solution for $\log_3(9)$

$\log_3(27)$	Rewrite as an exponential expression
$3^x = 27$	$3^3 = 27$ makes this true
$3^3 = 27$	Use the Log-Exponent Relationship
$\log_3(27) = 3$	Our solution for $\log_3(27)$

$\log_3(3)$	Rewrite as an exponential expression
$3^x = 3$	$3^1 = 3$ makes this true
$3^1 = 3$	Use the Log-Exponent Relationship
$\log_3(3) = 1$	Our solution for $\log_3(3)$

We can now rewrite our original expression as:

$$\log_3(9) + \log_3(27) - \log_3(3) = 2 + 3 - 1 = 4$$

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2. Applying the Product and Quotient Properties

Let's simplify and evaluate the same expression, but this time we will use the **product property of logs** and **quotient properties of logs**.

FORMULA TO KNOW

Product Property of Logs

$\log_b(xy) = \log_b(x) + \log_b(y)$

Quotient Property of Logs

$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

In our first example, we have some logarithms with the same base being added and subtracted. When we see addition, we can combine them into one logarithm by multiplying the arguments. Similarly, when we see subtraction, we can combine them into one logarithm using division.

EXAMPLE Evaluate $\log_3(9) + \log_3(27) - \log_3(3)$ using the Product Property and Quotient Property of Logs.

$\log_3(9)+\log_3(27)-\log_3(3)$	Use the Product Property of Logs to combine $\log_3(9)$ and $\log_3(27)$
$\log_3(9\cdot 27)-\log_3(3)$	Evaluate multiplication in parentheses
$\log_3(243)-\log_3(3)$	Use the Quotient Property of Logs to combine $\log_3(243)$ and $\log_3(3)$
$\log_3\left(\frac{243}{3}\right)$	Evaluate division in parentheses
$\log_3(81)$	Rewrite as an exponential expression
$3^x=81$	$3^4=81$ makes this true
$3^4=81$	Use the Log-Exponent Relationship
4	Our solution

3. Applying Multiple Properties

Recall other properties of logarithms:

FORMULA TO KNOW

Change of Base Property of Logs

$$\log_b(x)=\frac{\log_a(x)}{\log_a(b)}$$

Power Property of Logs

$$\log_b(x^n)=n\cdot\log_b(x)$$

Other Properties of Logs

$$\log_b(b)=1$$

$$\log_b(1)=0$$

Consider which of the above properties, along with the other properties in this lesson, you would need to simplify and evaluate the following logarithmic expression:

EXAMPLE Simplify and evaluate the logarithmic expression $\log_a\left(\frac{xz}{y^2}\right)$ when given the following:

$$\log_a(x)=3$$
$$\log_a(y)=5$$
$$\log_a(z)=-2$$

Inside the logarithm, we see multiplication, division, and an exponent. This tells us that we will likely use the Product, Quotient, and Power Properties. We are also given the values of $\log_a(x)$, $\log_a(y)$, and $\log_a(z)$, which will come in handy later in our evaluation.

$\log_a\left(\frac{xz}{y^2}\right)$	Apply the Product and Quotient Properties of Logs
$\log_a(x)+\log_a(z)-\log_a(y^2)$	Apply the Power Property of Logs
$\log_a(x)+\log_a(z)-2\log_a(y)$	Substitute the values $\log_a(x)=3$, $\log_a(y)=5$, $\log_a(z)=-2$
$3+(-2)-2(5)$	Evaluate multiplication
$3+(-2)-10$	Simplify
-9	Our solution

EXAMPLE Evaluate the logarithmic expression $\log_2\left(\frac{3x^2}{y}\right)$ when given the following:

$$\log(x)=1.7$$
$$\log(y)=0.53$$

Just like the above example, we see multiplication, division, and an exponent so we will again use the Product, Quotient, and Power Properties. However, notice the difference in bases. The expression we need to evaluate has a log of base 2, whereas the log values we are given are common logs, which have a base of 10. This means we'll need to use the Change of Base Property as well.

$\log_2\left(\frac{3x^2}{y}\right)$	Apply the Product and Quotient Properties of Logs
$\log_2(3)+\log_2(x^2)-\log_2(y)$	Apply the Power Property of Logs
$\log_2(3)+2\log_2(x)-\log_2(y)$	Apply the Change of Base Property
$\frac{\log(3)+2\log(x)-\log(y)}{\log(2)}$	Substitute the values $\log(x)=1.7$, $\log(y)=0.53$ and use calculator to evaluate $\log(3)$
$\frac{0.477+2(1.7)-0.53}{0.301}$	Simplify numerator
$\frac{3.347}{0.301}$	Divide
11.12	Our solution, rounded to the nearest hundredth

SUMMARY

When applying the log-exponent relationship, recall that a logarithmic equation can be written as an exponential equation. When **applying the product and quotient properties**, remember that the product property says that log base b of some product x and y is equal to log base b of x plus log base b of y. The quotient property says that log base b of some quotient x and y is equal to log base b of x minus log base b of y. The power property says that log base b of x to the nth power is equal to n times log base b of x. When evaluating a logarithmic expression, you may **apply multiple properties**.

FORMULAS TO KNOW

Change of Base Property of Logs

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Logarithmic Form to Exponential Form

$$\log_b(y) = x \leftrightarrow y = b^x$$

Other Properties of Logs

$$\log_b(b) = 1$$

$$\log_b(1) = 0$$

Power Property of Logs

$$\log_b(x^n) = n \cdot \log_b(x)$$

Product Property of Logs

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

Quotient Property of Logs

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

