Applying the Properties of Exponents

by Sophia Tutorial

WHAT'S COVERED

1. Properties of Exponents

There are several properties of exponents involving multiplication, division, and powers that can help us simplify expressions containing exponents. They are:

The Product Power: $a^n \bullet a^m = a^{n+m}$

The Quotient Power: $\frac{a^n}{a^m} = a^{n-m}$

The Power of Powers: $(a^n)^m = a^{nm}$

Power of a Product: $(ab)^n = a^n \bullet b^n$

Power of a Quotient: $(\frac{a}{b})^n = \frac{a^n}{b^n}$

Negative Exponents: $a^{-n} = \frac{1}{a^n}$

2. Cautions with Applying these Properties

There are a couple of things to watch out for when applying these properties. First, it is important to remember that the product and quotient powers can only be applied with the bases are the same. We cannot simply add the exponents if the bases are not the same.

HINT

$a^n \bullet b^m \neq (ab)^{n+m}$

Secondly, the product and quotient of power properties only work with multiplication and division; they do not work with addition or subtraction.

HINT

$(a + b)^n \neq a^n + b^n$

3. Applying the Properties of Exponents

Let's go through some examples where we can use different properties of exponents to simplify expressions:

EXAMPLE

$\frac{3x^2y^4}{xy^2}$	
$\frac{3xy^4}{y^2}$	Quotient Power: reduce power of x in the numerator and denominator
$3xy^2$	Quotient Power: reduce power of y by 2 in numerator and denominator
$3xy^2$	Our solution

If we see factors that appear in both the numerator and denominator, we cancel them out. This means that we reduce the power in both the numerator and denominator. If we cancel all factors in the denominator, the denominator is 1, and doesn't need to be written.

EXAMPLE

$\left(\frac{x^{-2}y^3}{y}\right)^{-1}$	
$(x^{-2}y^2)^{-1}$	Quotient Power: reduce power of y in the numerator and denominator
x^2y^{-2}	Power of Power: multiply each power by outside exponent, –1
$\frac{x^2}{y^2}$	Negative exponents: write in denominator with opposite power
$\left(\frac{x}{y}\right)^2$	Power of Quotient: write as a quotient squared

SUMMARY

Use the **properties of exponents** to simplify expressions and solve equations. However, consider the following **cautions when applying the properties**. You can only add or subtract the exponents in multiplication or division if the bases are the same. And you can only distribute an exponent across factors and not terms.

FORMULAS TO KNOW

Power of a Power Property of Exponents

$(a^n)^m = a^{nm}$

Power of a Product Property of Exponents

$(ab)^n = a^n \cdot b^n$

Power of a Quotient Property of Exponents

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Product Property of Exponents

$a^n \cdot a^m = a^{n+m}$

Property of Negative Exponents

$a^{-n} = \frac{1}{a^n}$

Quotient Property of Exponents

$\frac{a^n}{a^m} = a^{n-m}$