Applying the Properties of Radicals

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to simplify a radical expression using the properties of radicals. Specifically, this lesson will cover:

1. Properties of Radicals

There are several properties of radicals we can apply to simplify expressions involving radicals. The following properties are generally true whenever n is greater than 1, and a and b are both positive real numbers:

FORMULA TO KNOW

Properties of Radicals

$\sqrt[n]{a^n} = a$ and $(\sqrt[n]{a})^n = a$

Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$

Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Fractional Exponents: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

2. Cautions when Apply the Properties

Avoid these common errors when applying properties of radicals:

- The properties of radicals only apply to factors; they do not apply to terms. For example, we can use the product property of radicals to break $\sqrt{15}$ into two radicals $\sqrt{5} \cdot \sqrt{3}$ because $5 \cdot 3 = 15$. However, we cannot break $\sqrt{8}$ into $\sqrt{5}+\sqrt{3}$.

- We can only bring an exponent outside of a radical if it applies to everything underneath the radical. For example, we can rewrite $\sqrt[3]{x^2}$ as $(\sqrt[3]{x})^2$ because the exponent of 2 applied to everything underneath the radical. However, $\sqrt[3]{16x^2} \neq (\sqrt[3]{16x})^2$ This is because the exponent of 2 applies only to the *x*, not the 16. (We could rewrite the expression as $(\sqrt[3]{4x})^2$ because $16=4^2$.)
- Taking the odd-root of a negative number leads to a real number solution, because a negative value raised to an odd exponent is negative. However, taking the even-root of a negative value leads to a non-real solution, because a negative value raised to an even exponent is never negative.

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3. Applying the Properties of Radicals

When we recognize products, quotients, and powers with radicals, we can apply the properties of radicals to simplify the expression.

EXAMPLE

$\sqrt[3]{27xy^3}$	Use Product Property of Radicals
$\sqrt[3]{27}\cdot\sqrt[3]{x}\cdot\sqrt[3]{xy}$	Evaluate: $\sqrt[3]{27}=3$, $\sqrt[3]{y^3}=y$
$3\cdot\sqrt[3]{x}\cdot y$	Simplify
$3y\sqrt[3]{x}$	Our Solution

EXAMPLE

$\dfrac{\sqrt[3]{x^2y}}{\sqrt[3]{8xy^2}}$	Use Quotient Property of Radicals
$\sqrt[3]{\dfrac{x^2y}{8xy^2}}$	One factor of *x* cancels; one factor of *y* cancels
$\sqrt[3]{\dfrac{x}{8y}}$	Separate into two separate fractions
$\sqrt[3]{\dfrac{1}{8}\cdot\dfrac{x}{y}}$	Use Product Property of Radicals
$\sqrt[3]{\dfrac{1}{8}}\cdot\sqrt[3]{\dfrac{x}{y}}$	Use Quotient Property of Radicals and $\dfrac{x}{y}=xy^{-1}$
$\dfrac{\sqrt[3]{1}}{\sqrt[3]{8}}\cdot\sqrt[3]{xy^{-1}}$	Simplify
$\dfrac{1}{2}\sqrt[3]{xy^{-1}}$	Our Solution

SUMMARY

We can use the **properties of radicals** to simplify expressions and solve equations. There are some **cautions when applying the properties**. The properties of radicals apply only to factors, numbers, and variables combined by multiplication, not by addition or subtraction.

FORMULAS TO KNOW

Product Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Property of Fractional Exponents

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$