## Area

## by Sophia

## WHAT'S COVERED

In this lesson, you will learn how to calculate the area of a rectangle, square, and circle using formulas for area. Specifically, this lesson will cover:

1. Area Formulas
2. Calculate the Area Given Side Lengths or Radii
3. Calculate Side Lengths or Radii Given Area

## 1. Area Formulas

When we try to determine how much space is covered in a two-dimensional space, we need to use the concept of area. For example, when buying a house we oftentimes will consider the total living space in a house, which is considering how much floor space the house has.

Because area is represented in two dimensions, we represent quantities of area as squared units. For example, we might say that a square living room has a total area of 625 square feet, which might mean that the length of the room is 25 feet and the width is 25 feet.

## © BRAINSTORM

Can you think of another example where we might need to use area?

Suppose we wanted to paint the outer walls of a house, we may want to estimate the total area of the house's exterior to get an estimate of the total amount of paint that we would need.

## BIG IDEA

You will notice that when calculating area you will always be multiplying two distances with one another to determine the two-dimensional space covered by an object.
When calculating areas, there are a few common shapes that you will always come across, as we have previously defined. These shapes and their area formulas are listed below.

## Area of a Rectangle

$A=b h$, where $A$ is the area, $b$ is the base, and $h$ is the height.

$$
A=I W
$$



## $\backsim \quad$ HINT

Note that the area of a square is a special case of the area of a rectangle formula where the length and width are the same.

## $\int$ FORMULA TO KNOW

## Area of a Triangle

$$
A=\frac{1}{2} b h, \text { where } A \text { is area, } b \text { is base, and } h \text { is height. }
$$



Note that the height of a triangle is the distance of the line from one vertex (or corner) of the triangle to the opposite base, such that the line is perpendicular to the base.

## $\int$ FORMULA TO KNOW

## Area of a Circle

$$
A=\pi r^{2}
$$



## 『 HINT

Note that the radius of a circle is the distance from the center of a circle to the edge of the circle. Pi is a constant irrational number equal to 3.14159265 ....

## E TERM TO KNOW

Pi (п)
The ratio of a circle's circumference to its diameter; approximately equal to 3.14 .
Let's look at how we can use these different area formulas to find some unknown quantity.

## 2. Calculate the Area Given Side Lengths or Radii

$\Leftrightarrow$ EXAMPLE Suppose we are told that a rectangle has a length of 10 feet and a width of 12 feet. How would we find the rectangle's area?

$$
\begin{array}{rl}
\text { Area }=l \cdot w & I=10 \mathrm{ft}, w=12 \mathrm{ft} \\
\text { Area }=(10 \mathrm{ft})(12 \mathrm{ft}) & \text { Multiply length by width }
\end{array}
$$

$$
\begin{aligned}
& \text { Area }=120 \mathrm{ft} \cdot \mathrm{ft} \quad \text { Units of feet are also multiplied } \\
& \text { Area }=120 \mathrm{ft}^{2} \quad \text { Our Solution }
\end{aligned}
$$

Notice that when calculating area, we square the units of distance.
$\Leftrightarrow$ EXAMPLE Suppose we are asked to find the area of a circle with a diameter of 9 inches. How would we make this calculation? As with the previous example, we start by writing down the appropriate area formula, and substitute in the quantities we know.

Here, we are given the diameter of the circle, but we need to know the radius to use the formula. The diameter of a circle is simply the distance of the line passing through the center of a circle and touching the circle's edge. In other words, the diameter, $d$, is twice as long as the radius, $r$, $(d=2 r)$.

Since the diameter for this circle was 9 inches, the radius must be 4.5 inches.

$$
\begin{gathered}
\text { Area }=\pi r^{2} \quad \pi \approx 3.14, r=4.5 \text { in } \\
\text { Area }=\pi(4.5 \mathrm{in})^{2} \\
\text { Substitute } 4.5 \text { in for } r \\
\text { Area }=20.25 \pi \mathrm{in}^{2} \\
\text { Square the radius } \\
\text { Area }=63.6 \mathrm{in}^{2} \quad \text { Our Solution, rounded to the tenths place }
\end{gathered}
$$

## 3. Calculate Side Lengths or Radii Given Area

Sometimes, we may be given the area of an object and need to back-solve to find the measurements of a given part of an object. Let's look at some examples.
$\Leftrightarrow$ EXAMPLE If the area of a square is 400 square feet what is the length of the square?

When solving these types of problems, we follow the same process as we did before but then do some algebraic manipulation to solve the problem. Note that the length and width of a square is the same so we can refer to side length using a single variable, s.

$$
\begin{array}{cl}
400 \mathrm{sq} \mathrm{ft} & =\mathrm{s}^{2} \\
\sqrt{400 \mathrm{sq} \mathrm{ft}}=\sqrt{\mathrm{s}^{2}} & \text { Area }=400 \mathrm{sq} \mathrm{ft} \\
20 \mathrm{ft}=\mathrm{s} & \text { Our Solution }
\end{array}
$$

$\Leftrightarrow$ EXAMPLE If the area of a circle is $100 \pi \mathrm{ft}^{2}$, what is the radius of the circle?

Like in the previous example, we begin by substituting what we know into the appropriate formula. Notice that the area contains pi in it. This is often times the same when we represent area exactly; we leave the pi multiplied to the number.

$$
\begin{aligned}
100 \pi \mathrm{sqft}=\pi \mathrm{r}^{2} & \text { Area }=100 \pi \mathrm{sq} \mathrm{ft} \\
100 \mathrm{sqft}=r^{2} & \text { Divide both sides by } \pi \\
\sqrt{100 \mathrm{sqft}}=\sqrt{\mathrm{r}^{2}} & \text { Take the square root of both sides } \\
10 \mathrm{ft}=r & \text { Our Solution }
\end{aligned}
$$

## SUMMARY

As an introduction to area, we learned that area of an object is the amount of space enclosed in a two dimensional shape. Area is measured in square units, such as centimeters squared or inches squared. Common area formulas include areas of rectangles, triangles, and circles. We can also use formulas to calculate the area when given side lengths or radii, as well as to calculate the side lengths or radii when given an area.

Source: ADAPTED FROM "BEGINNING AND INTERMEDIATE ALGEBRA" BY TYLER WALLACE, AN OPEN SOURCE TEXTBOOK AVAILABLE AT www.wallace.ccfaculty.org/book/book.html. License: Creative Commons Attribution 3.0 Unported License

## 日 TERMS TO KNOW

$\mathrm{Pi}(\pi)$
The ratio of a circle's circumference to its diameter; approximately equal to 3.14.

## $\leftrightharpoons$ FORMULAS TO KNOW

Area of Circle

$$
A_{\text {circle }}=\pi r^{2}
$$

Area of Rectangle
$A_{\text {rectangle }}=b h$

Area of Triangle
$A_{\text {triangle }}=\frac{1}{2} b h$

