

# Basic Quadratic Factoring

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## WHAT'S COVERED

In this lesson, you will learn how to factor a quadratic expression in the form  $x^2 + bx + c$ . Specifically, this lesson will cover:

## 1. Binomial Multiplication

When factoring quadratic expressions, we are rewriting a quadratic from the form  $ax^2 + bx + c$  into the form  $(ax + p)(bx + q)$  (Note: the variables  $a$  and  $b$  are not necessarily equivalent in both forms: they just represent a variable number).

To better understand what we are looking for when we are factoring a quadratic, it is helpful to look at how we go in the opposite direction. This process is known as binomial multiplication and is modeled by FOIL: First, Outside, Inside, Last.

→ EXAMPLE Multiply  $(x + 2)(x - 3)$

$(x + 2)(x - 3)$  Multiply the first terms,  $x \cdot x = x^2$ , to start the calculation,

$x^2$  Multiply the outside terms,  $x \cdot -3 = -3x$ , and add to the calculation

$x^2 - 3x$  Multiply the inside terms,  $2 \cdot x = 2x$ , and add to the calculation

$x^2 - 3x + 2x$  Multiply the last terms,  $2 \cdot -3 = -6$ , and add to the calculation

$x^2 - 3x + 2x - 6$  Combine like terms,  $-3x$  and  $2x$

$x^2 - x - 6$  Our solution



### BIG IDEA

Look at the coefficient of the  $x$ -term and the constant term. When factoring a quadratic expression, we are looking for two integers:  $p$  and  $q$ . The sum of  $p$  and  $q$  is the coefficient of the  $x$ -term, and the product of  $p$  and  $q$  is the constant term.

In the example above, we can factor  $x^2 - x - 6$  as  $(x + 2)(x - 3)$ . We know this is true, because we FOILED

$(x+2)(x-3)$  to get  $x^2 - x - 6$ . Thinking about the relationship between factoring and FOIL, we see that the sum of 2 and -3 is -1 (the coefficient of  $x$ ), and the product of 2 and -3 is -6 (the constant term).

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## 2. Basic Quadratic Factoring

When factoring a basic quadratic expression in the form  $x^2 + bx + c$ , we need to identify two integers whose product is the constant term of the quadratic, and whose sum is the  $x$ -term coefficient. To identify these two integers, we identify pairs of factors that multiply to the constant term. From there, we add the integers in each pair together and work with the pair that sums to the  $x$ -term coefficient.

➔ **EXAMPLE** Factor  $x^2 + 8x + 15$ .

To begin, we will list pairs of integers that multiply to 15:

$p$	$q$
1	15
-1	-15
3	5
-3	-5

Note that it can include two negative numbers because the product will be positive. Next, we add  $p$  and  $q$ , looking for a sum of 8. Once we find our sum of 8, there is no point in finding the other sums, because we know we don't want to use those values for  $p$  and  $q$ .

$p$		$q$		sum
1	+	15	=	16
-1	+	-15	=	-16
3	+	5	=	8
-3	+	-5	=	

We have identified  $p$  as 3, and  $q$  as 5. This means that we can factor the quadratic as:

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

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## 3. The Sign of $pq$

Looking at the sign of the constant term can help you eliminate possible values for  $p$  and  $q$  when factoring. For instance, if the constant term is negative, we know that one of  $p$  or  $q$  must be negative, but not both (because two negative values result in a positive number when multiplied). The table below can guide you when thinking about the sign of  $pq$ .

	$q$	$-q$
$p$	$pq$	$-pq$
$-p$	$-pq$	$pq$

Here is how we can use the sign of  $pq$  to narrow our focus when factoring:

➞ EXAMPLE Factor  $x^2 - 7x + 12$ .

We see that the constant term is positive. This means that the signs of  $p$  and  $q$  must match: either both of them are positive numbers, or both of them are negative. Next, we look at the x-term. The coefficient is a negative number. This means that the sum of  $p$  and  $q$  must be negative. Since the sum of two positive numbers is always positive, we can conclude that  $p$  and  $q$  must be negative.

So we draft up pairs of negative numbers that equal positive 12 when multiplied:

$p$	$q$
-1	-12
-2	-6
-3	-4

Note that there is no need to write the other pairs of  $p$  and  $q$  that essentially switch what we already have, such as -4 and -3. Their sums will be the same because addition is commutative.

Now we add our integer pairs, and stop when we reach -7:

$p$		$q$		sum
-1	+	-12	=	-13
-2	+	-6	=	-8
-3	+	-4	=	-7

We have identified  $p$  as -3, and  $q$  as -4. This means that we can factor the quadratic as:

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

➞ EXAMPLE Factor  $x^2 - 7x - 18$ .

We see that the constant term is negative. This means that one of  $p$  or  $q$  must be negative, but not both. Next, we look at the x-term. The coefficient is a negative number. This means that the larger number will be negative and the smaller number will be positive.

So we draft up pairs of one positive number and one negative number that equals negative 18 when multiplied:

$p$	$q$
-18	1
-9	2
-6	3

Notice in each case, the larger number had the negative sign (18 is larger than 1, so the negative sign will go with the 18). Now we add our integer pairs, and stop when we reach a sum of -7:

$p$		$q$		sum
-18	+	1	=	-17
-9	+	2	=	-7
-6	+	3	=	

We have identified  $p$  as -9, and  $q$  as 2. This means that we can factor the quadratic as:

$$x^2 - 7x - 18 = (x - 9)(x + 2)$$



## SUMMARY

With **binomial multiplication**, we use the FOIL method. **Basic quadratic factoring** is the process of writing an equation from expanded form to factored form. Factoring involves finding two integers whose sum is the coefficient of the x-term, and whose product is the constant term of the quadratic equation. **The sign of  $pq$**  can help determine the two integers.

Source: ADAPTED FROM "BEGINNING AND INTERMEDIATE ALGEBRA" BY TYLER WALLACE, AN OPEN SOURCE TEXTBOOK AVAILABLE AT [www.wallace.ccfaculty.org/book/book.html](http://www.wallace.ccfaculty.org/book/book.html). License: Creative Commons Attribution 3.0 Unported License



## FORMULAS TO KNOW

### Basic Quadratic Factoring

$$(x + p)(x + q) = x^2 + (p + q)x + pq$$