Bayes' Rule

by Sophia

WHAT'S COVERED

This tutorial will cover the topic of Bayes' Rule. Our discussion breaks down as follows:

1. Bayes' Rule

Bayes' Rule is a theorem that allows us to turn around a conditional probability statement. This rule allows you to update or revise your probabilities in light of new information. Essentially, it reverses the conditional probability formula, allowing you to find the probability of A given B from the probability of B given A.

FORMULA TO KNOW

Bayes' Rule

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(B)}$$

TERM TO KNOW

Bayes' Rule

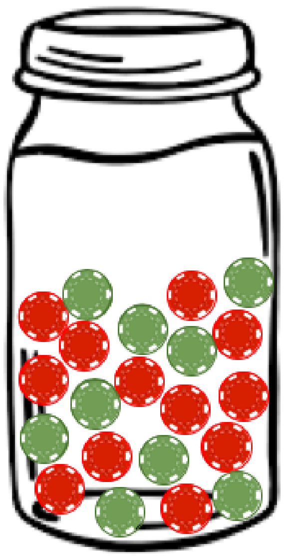
A reversal of the conditional probability formula that asks "given that this second event happened, what is the probability that this other event occurred first?"

1a. Game Show Example

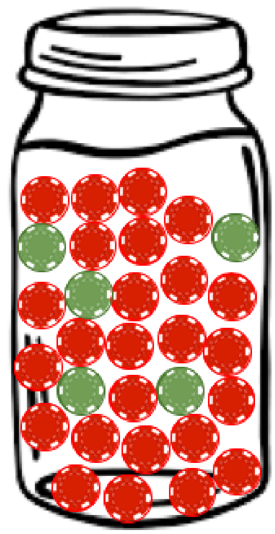
Suppose there's a famous television show called "Go For Broke.” On the show, a person gets to roll a die to determine which jar they're going to draw a colored chip from. If the chip is green, they will win a prize, and if the chip is red, they won’t win anything.

- If the contestant rolls a 1 or a 2, they select from jar A, which contains nine green chips and 12 red chips.

- If they roll a 3, 4, 5, or 6, they have to select from jar B, which only contains five green chips and 25 red.

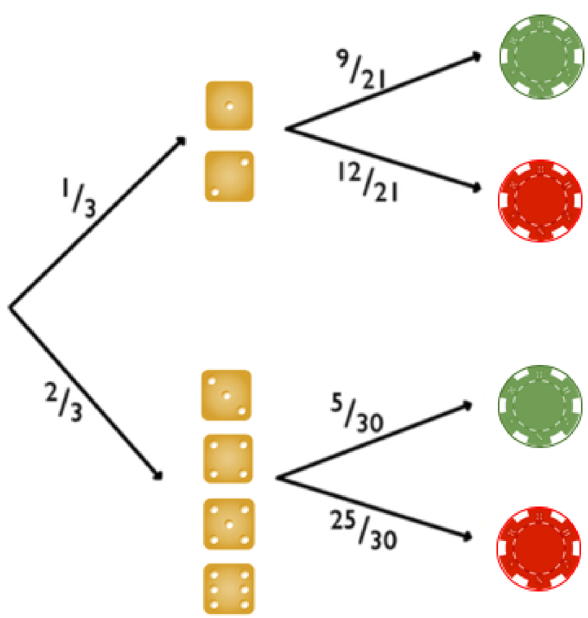


Roll a 1 or 2:
Pick from Jar A



Roll a 3, 4, 5, or 6:
Pick from Jar B

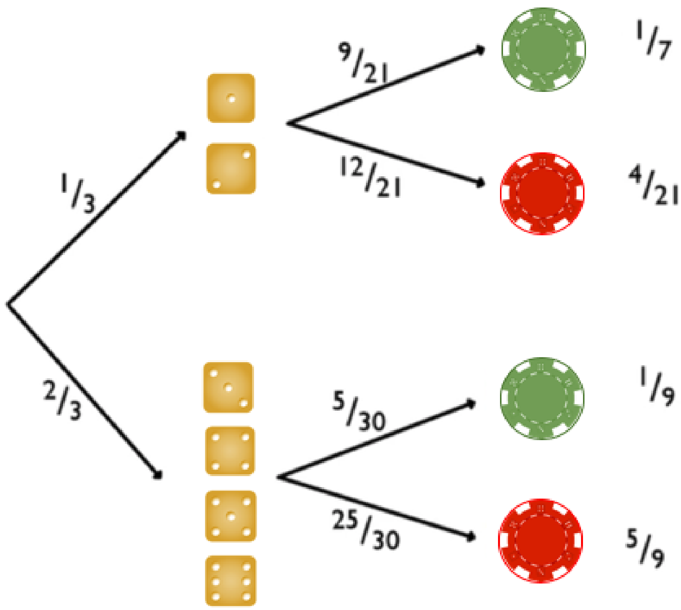
Rolling a 1 or a 2 and being able to select from jar A gives the contestant a better probability of winning a prize. Here is the tree diagram of the probabilities in this game:



- Contestants have a 1/3 probability of rolling 1 or 2 and selecting from Jar A.
  - If they are selecting from jar A, they have a 9/21 probability of getting the green chip and a 12/21 probability of getting the red chip.
- Contestants have a 2/3 probability of rolling 3, 4, 5, or 6 and selecting from Jar B.

- If they are selecting from jar B, they have a 5/30 probability of getting the green chip and a 25/30 probability of getting the red chip.

By multiplying out the values on the tree diagram, we find that we get these values for the probability of choosing either color chip from either jar:



THINK ABOUT IT

Suppose you tuned in late to *Go for Broke* one day and didn't get to see which jar the chip came from. All you see is that the contestant won the prize by getting the green chip.

Given they got a green chip, what's the probability that they rolled the 1 or 2, meaning they selected their winning chip from jar A?

To answer that question, what you're actually trying to find is the probability that you would have picked from A, given that you got the green chip. This requires the following conditional probability statement:

$$P(A \mid G)=\frac{P(A \text{ and } G)}{P(G)}$$

But recall that the "and" probability of two dependent events is the probability of one event times the conditional probability of the second event, given the first event occurred. So we can actually rewrite this equation:

$$P(A \mid G)=\frac{P(A \text{ and } G)}{P(G)}=\frac{P(A)\cdot P(G \mid A)}{P(G)}$$

When rewritten in this new formula, you may notice that the probability of G given A helps us to find the probability of A given G. When we actually go through and run the scenarios, the probability of A and G was one of the branches on the tree diagram, or 1/7.

To find the probability of G, we can see from the tree diagram that this can happen in two ways: selecting from jar A or jar B.

Plugging in this information, the equation simplifies down to the fraction 9/16::

$$P(A \mid G) = \frac{P(A) \cdot P(G|A)}{P(G)} = \frac{\frac{1}{7}}{\frac{1}{7}+\frac{1}{9}} = \frac{\frac{1}{7}}{16/63} = \frac{9/63}{16/63} = \frac{9}{16}$$

What this means is that out of every 16 times you pick the green, 9 of those times it came from jar A. This is a surprising conclusion! The probability that you would pick from jar A, given that you ended up winning, is actually over half.

1b. Finding a Crashed Plane

Another example of when you might use Bayes' theorem is during an actual army practice to find the wreckage of a plane.

Suppose that a plane was going from Miami to Mexico City, but then it crashed somewhere in the Gulf of Mexico. Here is a map of the scenario:

As the map shows, the plane may have lost radio contact in one location. Therefore, because they don't know what else might have happened with the plane, the search party might start searching in that area. However, perhaps a few days go by, and people find debris in a different location.

Bayes' theorem allows you to analyze the probability that the plane crashed in the region where they lost radio contact, given that the debris was found elsewhere, where the green circle is.

If that probability is low, you might choose to start searching in the green region, assuming that the plane tried to go around the storm by going north.

The idea is that we can update our guesses based on new information.

SUMMARY

Bayes' rule is a mathematical theorem that allows you to amend probability statements based on new information. With this rule, you can turn around the conditional probability statement and find the probability of A given B from the probability of B given A. Therefore, it lets you use newer knowledge to adjust probabilities of prior events.

Good luck!

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TERMS TO KNOW

Bayes' Rule

A reversal of the conditional probability formula that asks "given that this second event happened, what is the probability that this other event occurred first?"