Canceling Common Terms in Algebraic Fractions

by Sophia

WHAT'S COVERED

This tutorial covers how to cancel common terms in algebraic fractions, through the definition and discussion of:

1. Rational Expressions

A rational expression is a fraction whose numerator and denominator are polynomials. They are sometimes referred to as algebraic fractions. Reducing rational expressions is similar to reducing numerical fractions. Factors can be canceled *only* if they appear as factors of both the numerator and the denominator; they are canceled because they reduce to 1.

EXAMPLE You can see how the fraction below simplifies to 1.

$$\frac{(x+3)}{(x+3)} = 1$$

However, terms separated by addition or subtraction in the numerator or denominator cannot be canceled.

EXAMPLE Consider the expression below.

$$\frac{x-2}{x+5} \neq \frac{-2}{5}$$

You can verify this because if you substituted a value of 3 in for both xs, you'd have the following, which is *not* -2 over 5.

$$\frac{3-2}{3+5} = \frac{1}{8} \neq \frac{-2}{5}$$

2. Simplifying Fractions

In review, you can simplify fractions by canceling common factors in the numerator and denominator.

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STEP BY STEP

1. Write the numerator and denominator as products of prime factors:
2. Cancel factors that appear in both the numerator and denominator.
3. Multiply remaining factors in the numerator and denominator.

EXAMPLE Suppose you want to simplify the following fraction.

$$\frac{48}{90}$$

Step 1: Write the numerator and denominator as products of prime factors:

$$\frac{48}{90}=\frac{2\cdot 2\cdot 2\cdot 2\cdot 3}{2\cdot 3\cdot 3\cdot 5}$$

Step 2: Cancel factors that appear in both the numerator and denominator. You can cancel out one 2 and one 3 in both the numerator and denominator.

$$\frac{48}{90}=\frac{\cancel{2}\cdot 2\cdot 2\cdot 2\cdot \cancel{3}}{\cancel{2}\cdot \cancel{3}\cdot 3\cdot 5}$$

Step 3: Multiply remaining factors in the numerator and denominator. In the numerator, you have three 2s remaining, and in the denominator, you have one 3 and one 5, so multiply accordingly and simplify.

$$\frac{48}{90}=\frac{\cancel{2}\cdot 2\cdot 2\cdot 2\cdot \cancel{3}}{\cancel{2}\cdot \cancel{3}\cdot 3\cdot 5}=\frac{2\cdot 2\cdot 2}{3\cdot 5}=\frac{8}{15}$$

DID YOU KNOW

Simplifying rational expressions is similar to reducing numerical fractions, because you identify common factors to cancel.

3. Greatest Common Factor in Polynomials

Finding the greatest common factor of a polynomial is a helpful strategy when simplifying algebraic fractions in which common factors appear in both the numerator and denominator.

EXAMPLE Suppose you want to factor the expression:

$$4x^3-8x$$

You can start by writing each term as a product of factors.

$$4x^3-8x=2\cdot 2\cdot x\cdot x\cdot x-2\cdot 2\cdot 2\cdot x$$

You can see that both terms have two 2s and one x in common. Multiplying these common factors together equals 4x.

$$4x^3 - 8x = 2\cdot 2\cdot x\cdot x\cdot x - 2\cdot 2\cdot 2\cdot x$$

You can begin to rewrite your expression by writing your greatest common factor, 4x, on the outside of the parentheses, and writing the remaining factors of each term inside the parentheses. From the first term, you have two xs remaining, or x^2, and in your second term, you have a negative and a 2, in other words, -2. This results in the factored form of your expression.

$$4x(x^2-2)$$

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4. Simplifying Rational Expressions

When simplifying rational expressions, you want to start by writing both the numerator and denominator as a products of their prime factors and variable factors.

↱ EXAMPLE Suppose you want to simplify the following expression.

$$\frac{12a^6}{8a^4}$$

Begin by rewriting both numerator and denominator as products of their prime factors and variable factors: $\frac{12a^6}{8a^4} = \frac{2\cdot 2\cdot 3\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a}{2\cdot 2\cdot 2\cdot a\cdot a\cdot a\cdot a}$

You can see that both the numerator and denominator have two 2s and four as as common factors. You can cancel all of these factors out.

$$\frac{12a^6}{8a^4} = \frac{\cancel{2}\cdot\cancel{2}\cdot 3\cdot\cancel{a}\cdot\cancel{a}\cdot\cancel{a}\cdot\cancel{a}\cdot a\cdot a}{\cancel{2}\cdot\cancel{2}\cdot 2\cdot\cancel{a}\cdot\cancel{a}\cdot\cancel{a}\cdot\cancel{a}}$$

This leaves you with one 3 and two as multiplied together in the numerator, and one 2 in the denominator. Multiplying your remaining factors provides:

$$\frac{3\cdot a\cdot a}{2} = \frac{3a^2}{2}$$

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TRY IT

Consider the following expression.

$$\frac{7x+21}{2x+6}$$

Simplify this expression.

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Begin by factoring both the numerator and denominator.

$$\frac{7x+21}{2x+6}=\frac{7\cdot x+7\cdot 3}{2\cdot x+2\cdot 3}$$

Looking at your numerator, you can see that you have a common factor of 7. Therefore, you can factor out a 7 and rewrite your expression with the remaining factors, x and plus 3. Next, looking at your denominator, you can see that you have a common factor of 2 in both terms, so, again, you can factor out the 2 by writing it on the outside of the parentheses and writing your remaining factors, x and plus 3, inside the parentheses. $\frac{7x+21}{2x+6}=\frac{7\cdot x+7\cdot 3}{2\cdot x+2\cdot 3}=\frac{7(x+3)}{2(x+3)}$

Now you can see that you have a common factor of x plus 3 in both the numerator and the denominator, which can be canceled out, leaving you with your final expression:

$$\frac{7x+21}{2x+6}=\frac{7\cdot x+7\cdot 3}{2\cdot x+2\cdot 3}=\frac{7(x+3)}{2(x+3)}=\frac{7\cancel{(x+3)}}{2\cancel{(x+3)}}=\frac{7}{2}$$

SUMMARY

Today you learned the definition of a **rational expression**, or algebraic fraction, which is a fraction whose numerator and denominator are polynomials. You also reviewed how to **simplify fractions** and find the **greatest common factor in polynomials**. Lastly, you learned how to **simplify rational expressions**, by 1) writing the numerator and denominator as products of prime factors, 2) canceling factors that appear in both the numerator and denominator, and 3) multiplying the remaining factors in the numerator and denominator.

Source: This work is adapted from Sophia author Colleen Atakpu.

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