# Center and Variation of a Sampling Distribution

*by Sophia*

WHAT'S COVERED

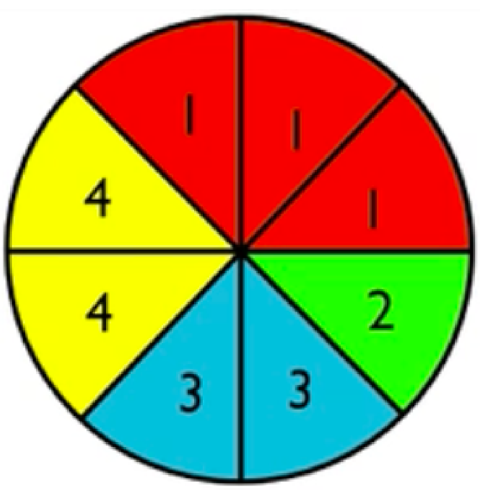
This tutorial will explain how to find the mean and standard deviation of a sampling distribution of the sample means. Our discussion breaks down as follows:



## 1. Mean of a Distribution of Sample Means

In this tutorial, you're going to learn about the center and variation of a sampling distribution. We're going to be using the mean to measure center, and the standard deviation to measure the variation.

Suppose you have a spinner with the following sectors:

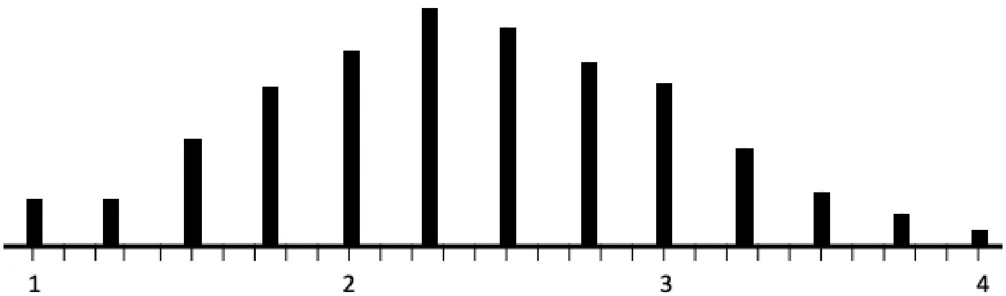



It's fairly easy to find the mean number spun from one spin. You just add up all the values, and divide by 8 because they're all equally likely sectors:

$$Mean = \frac{1+1+1+2+3+3+4+4}{8} = \frac{19}{8} = 2.375$$

You end up with 2.375 as your mean. Using the standard deviation formula or Excel, the population standard deviation of the spinner is 1.218.

Now suppose you spun it four times to obtain an average. Next, you did it again and obtained another average, and then another average. You could eventually consider every possible set of four outcomes. If you consider every possible scenario and plot each scenario on a graph like the one below, you can create what's called a sampling distribution of sample means.




For this distribution, it looks like the data average is somewhere around the 2.25 or 2.5 region. Those are the most likely averages from four spins.

Now, this distribution of sample means *itself* has a mean. In the case of this distribution of sample means, the mean is somewhere in the center. In fact, the number is 2.375, which is the same as the mean of the original spinner. So, the **mean of the distribution of sample means** is the same as the mean of the original distribution for the spinner. Sometimes we call this the parent distribution or the grand mean.

Symbolically it looks like this:

FORMULA TO KNOW

**Mean of a Distribution of Sample Means**

$\mu_{\bar{x}} = \mu_{original}$

TERM TO KNOW

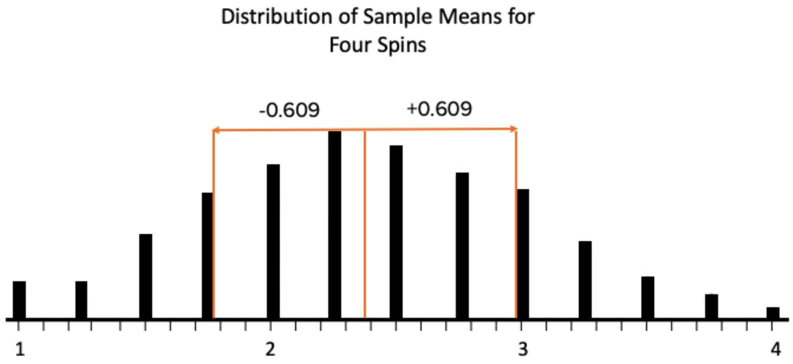
**Mean of a Distribution of Sample Means**

The average of all possible means from all possible samples of a given size. It will be equal to the mean of the original population.

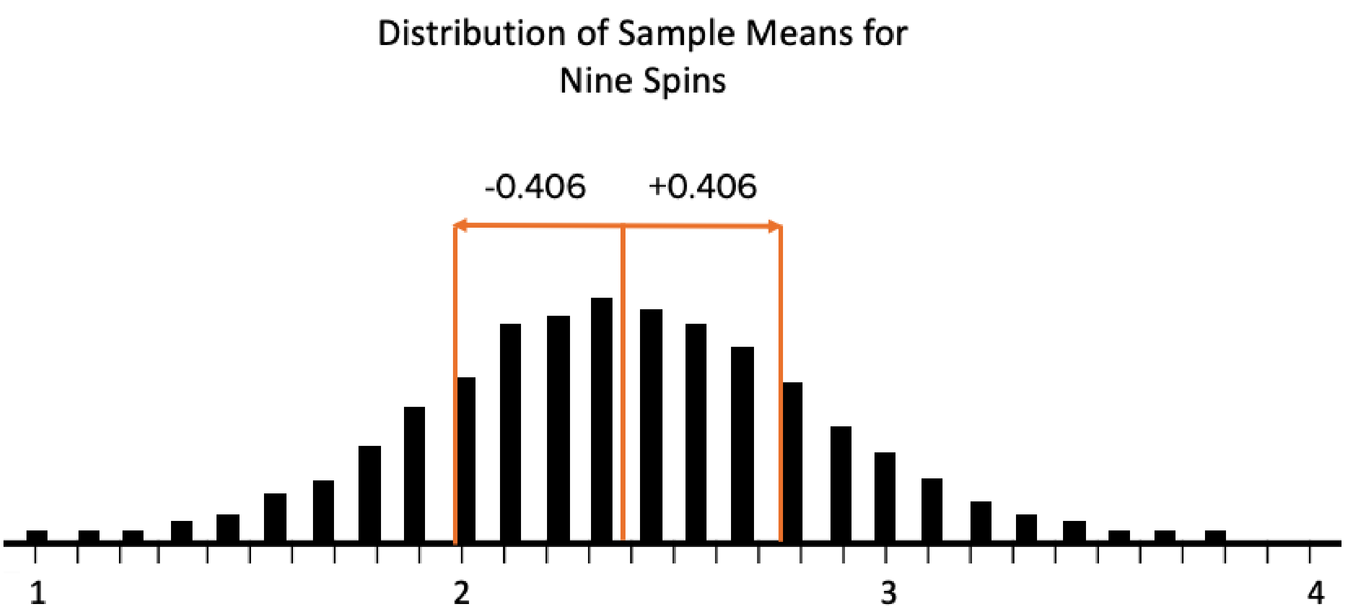
# 2. Standard Deviation of a Distribution of Sample Means

Recall that the standard deviation of the original distribution for the spinner was 1.218. There is also a **standard deviation of the distribution of sample means**, shown below for 4 spins. The standard deviation, in this case, is 0.609.

How does that 0.609 relate to 1.218? Well, 0.609 is half of 1.218. The standard deviation on the distribution for four spins is only half as large as the original standard deviation was.




What is the distribution of sample means when you spin nine times? You'll notice that the mean of the distribution for nine spins is the same as the other means: 2.375. Next, look at the shape. You can see that the extreme values are much less likely now, and things start moving towards the center. This might be indicative of the standard deviation getting smaller yet again. In fact, the standard deviation is just 0.406 on either side of the mean.




Let's consider the three cases so far:

- When you spun just once, the mean was 2.375, and the standard deviation was 1.218.
- When you spun four times, the mean was 2.375. Standard deviation was 0.609, which is half of the standard deviation of the original distribution.

- When you spun nine times, again the mean was 2.375. Standard deviation was 0.406, which is a third of the standard deviation of the original distribution.

As the number of spins increases, the standard deviation goes down. However, it's not linear; it's proportional to the inverse of the square root of n. Therefore, to calculate the standard deviation of a distribution of a sample means, you divide the original standard deviation by the square root of sample size.

TRY IT

A phone manufacturer claims their devices can play 8 hours of video on a full charge with a standard deviation of 0.2 hours. As part of a online review website, you test a sample of 4 phones.

**Calculate the expected mean and standard deviation of your sampling distribution.** +

Your mean will be the same as your population mean, 8 hours.

The standard deviation can be found with this formula; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

0.2/√4 = 0.1 hours

FORMULA TO KNOW

**Standard Deviation of a Distribution of Sample Means**

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

TERM TO KNOW

**Standard Deviation of a Distribution of Sample Means**

The standard deviation of all possible means from all possible samples of a given size. It will be equal to the standard deviation of the original population, divided by the square root of the sample size.

SUMMARY

The mean of a sampling distribution, from samples of size n, is always going to be the same as the mean of the original distribution it came from, or the parent distribution. The mean of all the x-bars is the same as the original mean from the parent distribution. The standard deviation, on the other hand, gets smaller as the sample size increases. The larger the sample size, the more likely it is that the extreme values will get evened out and pulled back towards the mean. Thus, the standard deviation decreases, and you can quantify the decrease in standard deviation. The standard deviation for the sampling distribution is the standard deviation of the parent distribution, divided by the square root of sample size.

Good luck!

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## TERMS TO KNOW

**Mean of a Distribution of sample means**

The average of all possible means from all possible samples of a given size. It will be equal to the mean of the original population.

**Standard Deviation of a Distribution of sample means**

The standard deviation of all possible means from all possible samples of a given size. It will be equal to the standard deviation of the original population, divided by the square root of the sample size.

## FORMULAS TO KNOW

**Mean of a Distribution of sample means**

$\mu_{\bar{x}} = \mu_{original}$

**Standard Deviation of a Distribution of sample means**

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$