Checking Your Solution

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WHAT'S COVERED

This tutorial covers checking your solution when solving an equation, through the discussion of:

1. Assessing the Reasonableness of a Solution

When solving math problems, it's a good idea to check that your answer is reasonable or correct. Checking to make sure the answer is reasonable does not necessarily mean that it is correct, but rather that it makes logical sense.

> EXAMPLE Suppose you want to find 20% of $50. The correct solution is $10, but common mistakes could lead you to find solutions of $100 or $1,000. Therefore, a *reasonable* answer may be $12 or $13.

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2. Checking the Solution to an Equation

You can verify a solution when solving an equation by plugging the solution back into the equation to get a true statement.

> EXAMPLE Evaluate $3(x+2)-8=-x+6$.

$3(x+2)-8=-x+6$	Our Equation
$3x+6-8=-x+6$	Start by distributing the 3 on the outside of the parentheses.
$3x-2=-x+6$	Next, combine like terms, 6 and minus 8, which equals negative 2.
$\begin{array}{rcl} 3x-2 &=& -x+6 \\ +x & & +x \end{array}$	From there, add x to both sides.
$\begin{array}{rcl} 4x-2 &=& 6 \\ +2 & & +2 \end{array}$	Then, add 2 to both sides.
$\frac{4x}{4}=\frac{8}{4}$	Finally, divide each side by 4.
$x=2$	Our Solution

Now that we have our solution, we can verify it by substituting 2 in the equation for x:

$$3(2+2)-8 = -2+6$$
$$3(4)-8 = -2+6$$
$$12-8 = -2+6$$
$$4 = 4$$

However, what happens when your solution is incorrect?

EXAMPLE What if you solved the equation in the manner shown below, and incorrectly combined 6 minus 8 on the left side to get -14? Ultimately, you'd arrive at an incorrect solution of x equals 5.

$$3x+6-8 = -x+6$$
$$3x-14 = -x+6$$
$$3x+x-14 = -x+x+6$$
$$4x-14 = 6$$
$$4x-14+14 = 6+14$$
$$4x = 20$$
$$\frac{4x}{4} = \frac{20}{4}$$
$$x = 5$$

In the same way that you verified the correct solution above, consider what happens when you attempt to verify your incorrect solution of x equals 5, by substituting 5 into your equation for x.

$$3(5)+6-8 = -(5)+6$$
$$15+6-8 = -5+6$$
$$13 \neq 1$$

Simplify 5 plus 2 in the parentheses, which equals 7. Multiply by 3 to give you 21. Finally, subtract 8 to arrive at the simplified answer of 13 on the left side of the equation. Simplifying on the right side, -5 plus 6, equals 1. Your final equation is 13 equals 1, which is a false statement. Therefore, your solution of x equals 5 is incorrect.

BIG IDEA

When you find that your solution is incorrect, you need to review your steps in solving the equation to see where you went wrong. Make sure to be careful, because it's also possible to find a correct answer when solving the equation, but to make a mistake while checking the answer. Therefore, it helps to double-check before assuming a mistake was made while trying to solve the equation.

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3. Checking the Solution to an Inequality

You can also verify a solution when solving an inequality in the same way you check a solution to an equation.

EXAMPLE Evaluate $5x < -(x+6)$.

$5x < -(x+6)$	Our Inequality
$5x < -x-6$	Start by distributing the negative to the terms in the parentheses.

$5x < -x-6$ $+x \quad +x$	Add x on both sides.
$\frac{6x}{6} < \frac{-6}{6}$	Divide both sides by 6.
$x < -1$	Our Solution

Next, to test our solution set, we will choose a value for x within our solution set, such as -2, because -2 is less than -1. Substituting negative 2 in the inequality and simplifying on both the left and the right side provides:

$$5(-2) < -(-2+6)$$
$$-10 < -4$$

As you can see, the final expression is negative 10 is less than negative 4, which is a true statement, so your solution is correct.

However, the same process of solving and verification applies even if your solution is incorrect. Any statements that are false but that you expect to be true, or any statements that are true but that you expect to be false, indicate an error. You can test these statements in the following example by using an incorrect solution.

↱ EXAMPLE Suppose that during the initial distribution when solving the equation, you neglect to distribute the negative to both terms in the parentheses.

$$5x < -(x+6)$$
$$5x < -x+6$$
$$6x < 6$$
$$x < 1$$

Now, the solution set, x is less than 1, tells you that all values within the set will satisfy the inequality, while other values will not. Therefore, you would pick some values to test your solution set. Suppose you start with x equals -10. You expect this to be true, because -10 is less than 1; therefore, you expect to have a true statement.

$$5(-10) < -(-10+6)$$
$$-50 < -(-4)$$
$$-50 < 4$$
$$\textit{TRUE}$$

Substituting -10 in for x in the equation simplifies to -50 is less than 4, which is a true statement. So far, so good!

Next, try another point, x equals 2. You expect this to be false because 2 is *not* less than 1, so you expect to have a false statement.

$$5(2) < -(2+6)$$
$$10 < -8$$
$$\textit{TRUE}$$

Substituting 2 in for x in your inequality simplifies to 10 is less than -8, which is a false statement, as you expected.

However, you may still have an error, so try one other point, x equals 0. You expect this to yield a true statement because 0 is less than 1.

$$5(0) < -(0+6)$$
$$0 < -6$$
$$\textit{FALSE}$$

Your final expression is 0 is less than negative 6, which is a false statement, leading you to believe that there was an error made. In this case, you would then go back and check your original work in solving the inequality.

SUMMARY

Today you learned that when solving math problems, it's a good idea to check your solution. You learned that one step in checking your solution is to **assess the reasonableness of your solution**, which does not necessarily mean that it is correct, but that it makes logical sense. You also learned that **checking a solution when solving an equation or inequality** involves plugging the solution back into the equation or inequality to get a true statement.

Source: This work is adapted from Sophia author Colleen Atakpu.

