Completing the Square

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WHAT'S COVERED

In this lesson, you will learn how to determine an equivalent quadratic equation by completing the square. Specifically, this lesson will cover:

1. Solving a Quadratic in the Form $(x+a)^2$

Some quadratic equations are difficult to solve, and some quadratic equations are easy to solve. Depending on how the equation is provided, certain methods can be very straightforward.

EXAMPLE Find the solutions for quadratic equation $(x+3)^2=25$.

We can find solutions to x by performing inverse operations, just like we solve other multi-step equations:

$(x+3)^2=25$	Take square root of both sides
$x+3=\pm\sqrt{25}$	Simplify square root
$x+3=\pm 5$	Subtract 3 from both sides
$x=-3\pm 5$	Create two solutions, one with addition and one with subtraction
$x=-3-5,\ \ x=-3+5$	Evaluate
$x=-8,\ \ x=2$	Our solutions

Unfortunately, not all quadratic equations come in this form. However, we can manipulate the equation to write it as such. This requires a process known as completing the square.

2. FOIL & Completing the Square

To understand the mechanics of **completing the square**, it is helpful to connect it to the FOIL process. Let's take the general expression $(x+a)^2$ and FOIL it:

$(x+a)^2$	Multiply two factors of $(x+a)$

$(x+a)(x+a)$	FOIL
$x^2+ax+ax+a^2$	Combine like terms
$x^2+2ax+a^2$	Our solution

BIG IDEA

The coefficient of the x-term is $2a$, and the constant term is a^2. If we can manipulate an expression to be in the form $x^2+2ax+a^2$, then we can write it equivalently as $(x+a)^2$. This is the goal of completing the square.

TERM TO KNOW

Completing the Square

The process of converting a quadratic equation in the form $ax^2+bx+c=0$ into an expression involving a perfect square trinomial.

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3. Solving a Quadratic by Completing the Square

The process of completing the square follows a specific set of steps in order to convert the equation into one similar to our very first example:

STEP BY STEP

1. Move the constant term to the other side of the equation.
2. Divide the entire equation by the x-square coefficient.
3. Separately, divide the x-term coefficient by two, then square it.
4. Add this quantity to both sides of the equation.
5. Write one side of the equation from expanded form into factored form, expressed as a single factor squared.

Let's apply these steps to the following example.

EXAMPLE Rewrite $2x^2-12x-14=0$ by completing the square.

$2x^2-12x-14=0$	Move the constant term, 14, to the other side by adding 14 to both sides
$2x^2-12x=14$	Divide entire equation by the x-square coefficient, 2
$x^2-6x=7$	Separately, divide the x-term coefficient, 6, by two, then square it
$-6 \rightarrow \frac{-6}{2} = -3 \rightarrow (-3)^2=9$	Add this value, 9, to both sides
$x^2-6x+9=7+9$	Simplify the right side
$x^2-6x+9=16$	Rewrite left side as binomial squared

$(x-3)^2 = 16$ Equivalent equation to $2x^2 - 12x - 14 = 0$

The last step is a bit tricky. We were able to write the right side of the equation as a binomial squared, because of the relationship between the coefficients and the constant term. When -3 is doubled, it is equivalent to the x-term coefficient in expanded form, -6, AND when it is squared, it equals to constant term in the expanded form, 9.

Now that we have converted an equation from standard form into a binomial squared, we can solve this equation following the same procedure as our very first example:

$(x-3)^2 = 16$	Take square root of both sides
$x - 3 = \pm\sqrt{16}$	Simplify square root
$x - 3 = \pm 4$	Add 3 to both sides
$x = 3 \pm 4$	Create two solutions, one with addition and one with subtraction
$x = 3 - 4, \quad x = 3 + 4$	Evaluate
$x = -1, \quad x = 7$	Our solutions

SUMMARY

When **solving a quadratic in the form (*x* + *a*)2**, we can find the solutions by performing inverse operations. **FOIL and completing the square** is a method of solving a quadratic equation. **Solving a quadratic by competing the square** involves rewriting a quadratic equation as a perfect square trinomial so that it can be written in factored form as $(x+a)^2$, where a and *y* are real numbers.

TERMS TO KNOW

Completing the Square

The process of converting a quadratic equation in the form $ax^2 + bx + c = 0$ into an expression involving a perfect square trinomial.