Complex Numbers in Electrical Engineering

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WHAT'S COVERED

In this lesson, you will learn how to calculate the voltage of a circuit given the current and resistance. Specifically, this lesson will cover:

1. Complex Numbers

A complex number is a number in the form $a + bi$, containing both a real part and an imaginary part. The imaginary part is followed by i , which is the imaginary unit, $\sqrt{-1}$.

One application of complex numbers is in electrical engineering (as well as other engineering and scientific fields). Complex numbers occur in calculations involving electrical currents, which will be explored in the examples below. Depending on the situation, we will need to either multiply or divide two complex numbers. During these processes, we use FOIL and complex conjugates to find our solutions. Let's briefly review the FOIL process and complex conjugates.

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2. FOIL & Complex Conjugate Review

FOIL stands for First, Outside, Inside, Last, and refers to the terms that are multiplied together to form individual addends to the product.

↪ EXAMPLE Multiply $(x+2)(x-3)$.

$(x+2)(x-3)$	Multiply first terms: $x \cdot x = x^2$
x^2	Multiply outside terms: $x \cdot -3 = -3x$
x^2-3x	Multiply inside terms: $2 \cdot x = 2x$
$x^2-3x+2x$	Multiply last terms: $2 \cdot -3 = -6$
$x^2-3x+2x-6$	Combine like terms

$x^2 - x - 6$ Our solution

HINT

When using FOIL with two complex numbers, one of our terms will be an i^2 term. This simplifies to a real number because $i^2 = -1$

When dividing two complex numbers, we use the denominator's complex conjugate to create a problem involving fraction multiplication. A complex number and its conjugate differ only in the sign that connects the real and imaginary parts. Here is a table of complex numbers and their complex conjugates.

Complex Number	Complex Conjugate
$8+4i$	$8-4i$
$-6+3i$	$-6-3i$
$7-5i$	$7+5i$
$-2-9i$	$-2+9i$

HINT

We use the denominator's complex conjugate to create a fraction equivalent to 1. As we will see in our division example, this eliminates all imaginary numbers from the denominator.

3. Voltage, Current, and Resistance

When working with electrical circuits, electrical engineers often apply the following formula to relate voltage, current, and resistance:

$V = I \cdot R$, where $V = \text{voltage}$, $I = \text{current}$, $R = \text{resistance}$

The voltage is measured in volts, the current is measured in amps, and the resistance is measured in ohms.

HINT

The notation engineers use for complex numbers is a bit different than what we may be used to seeing. There are generally two big differences:

- Engineers commonly use j instead of i , so as not to confuse the imaginary unit with the variable for current. So keep in mind in these examples that whenever we see j , this represents our imaginary unit, and has a value of $j = \sqrt{-1}$.
- In addition to using j , this variable is also often written before its coefficient, rather than after. For example the complex $2+3i$ number might be written as $2+j3$.

4. Multiplication using Voltage, Current, and Resistance

If we are finding the voltage, *V*, we will multiply the current, *I*, by the resistance, *R*.

↪ EXAMPLE An electrical circuit has a current of $3-j3$ amps, and a resistance of $2+j5$ ohms. What is the voltage of the circuit?

To find the voltage, we need to multiply the current by the resistance, giving us the equation:

$$V = I \cdot R$$
$$V = (3-j3)(2+j5)$$

Recall that j and i are interchangeable, so we can replace all instances of j with i when multiplying. So $3-j3$ can be written as $3-3i$ and $2+j5$ can be written as $2+5i$.

$$V = (3-3i)(2+5i)$$

We can find the product of current and resistance by using FOIL:

$V = (3-3i)(2+5i)$	Multiply first terms: $3 \cdot 2 = 6$
$V = 6$	Multiply outside terms: $3 \cdot 5i = 15i$
$V = 6+15i$	Multiply inside terms: $-3i \cdot 2 = -6i$
$V = 6+15i-6i$	Multiply last terms: $-3i \cdot 5i = -15i^2$
$V = 6+15i-6i-15i^2$	Combine like terms
$V = 6+9i-15i^2$	Rewrite $-15i^2$ as +15
$V = 6+9i+15$	Combine like terms
$V = 21+9i$	Our solution

We can express the voltage as: $21+j9$ volts.

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5. Division using Voltage, Current, and Resistance

Suppose we need to find the current, *I*. We can divide the voltage, *V*, by the resistance, *R*.

↪ EXAMPLE An electrical circuit has a voltage of $22+j3$ volts, and a resistance of $5+j2$ ohms. What is the circuit's current?

Here, we will need to divide the voltage by the resistance in order to get an expression for the current:

$$I = \frac{V}{R}$$
$$I = \frac{22+j3}{5+j2}$$

Remember to rewrite this using i instead of j :

$$I=\frac{22+3i}{5+2i}$$

To solve complex number division problems, we multiply the fraction by another fraction equivalent to 1, with the denominator's complex conjugate as the numerator and denominator of the second fraction:

$I=\frac{22+3i}{5+2i}$	Multiply by a second fraction with the conjugate $5-2i$ in the numerator and denominator
$I=\frac{22+3i}{5+2i}\cdot\frac{5-2i}{5-2i}$	Multiply the two fractions
$I=\frac{(22+3i)(5-2i)}{(5+2i)(5-2i)}$	Use FOIL to evaluate numerator and denominator
$I=\frac{110-44i+15i-6i^2}{25-10i+10i-4i^2}$	Combine like terms in numerator and denominator
$I=\frac{110-29i-6i^2}{25-4i^2}$	Rewrite $-6i^2$ as $+6$ and $-4i^2$ as $+4$
$I=\frac{110-29i+6}{25+4}$	Combine like terms in numerator and denominator
$I=\frac{116-29i}{29}$	Separate into two fractions
$I=\frac{116}{29}-\frac{29i}{29}$	Simplify fractions
$I=4-i$	Our solution

The circuit has a current of $4-j$ amps.

SUMMARY

A **complex number**, a plus bi, contains a real part, a, and an imaginary part, *b*, and the imaginary unit, *i*. **Reviewing FOIL and conjugates**, the conjugate of a binomial is a binomial with the opposite signs between its terms.

Electrical engineers often use complex numbers when working with the equation relating **voltage, resistance, and current**. Engineers and scientists often use the letter *j* to refer to the imaginary number *i*, so as not to confuse lowercase *i* with uppercase *I*, which is the variable for current. FOIL is used when solving **multiplication using *V* equals *I*•*R*** and conjugates are used when solving **division using *V* equals *I*•*R***, so that the denominator has no imaginary numbers.