Compound Inequalities

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WHAT'S COVERED

In this lesson, you will learn how to solve an "AND" compound inequality. Specifically, this lesson will cover:

1. "OR" Inequalities

The first type of a compound inequality is an "OR" inequality. For this type of inequality we want a true statement from either one inequality OR the other inequality OR both. When we are graphing these type of inequalities we will graph each individual inequality above the number line, then move them both down together onto the actual number line for our graph that combines them together.

When we give interval notation for our solution, if there are two different parts to the graph we will put a ∪ (union) symbol between two sets of interval notation, one for each part.

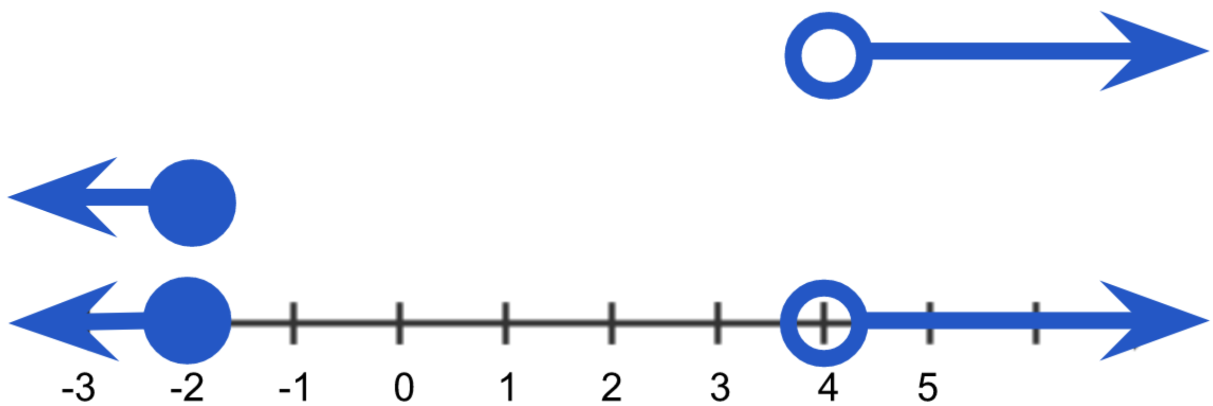
EXAMPLE Suppose the value of *x* falls under $2x-5>3$ or $4-x\geq 6$. Solve each inequality, graph the solution, and give interval notation of the solution.

$\begin{array}{r} 2x-5>3 \\ \underline{+5 \quad\; +5} \end{array}$	Solve by first adding 5 to both sides.
$\dfrac{2x}{2}>\dfrac{8}{2}$	Divide both sides by 2
$x>4$	Our solution
$\begin{array}{r} 4-x\geq 6 \\ \underline{-4 \quad -4} \end{array}$	Solve by first subtracting 4 from both sides.

$$\frac{-x}{-1} \geq \frac{2}{-1}$$ Divide both sides by -1. Don't forget to flip the sign!

$x \leq -2$ Our solution

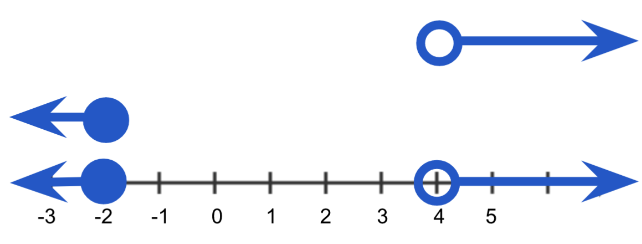
Graph the inequalities separately above a number line and then combine for a final graph.

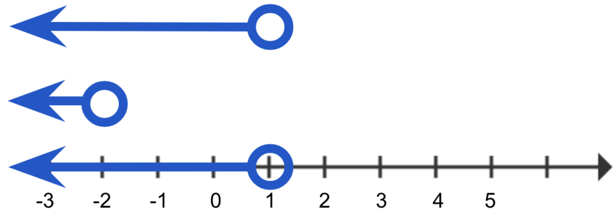
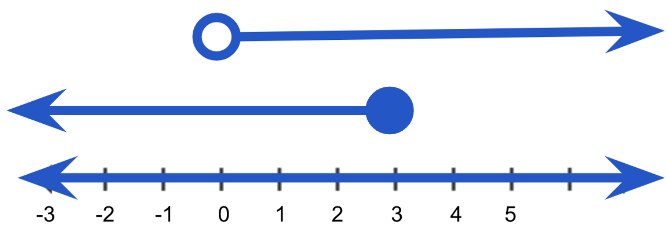


Interval notation: $(-\infty, -2] \cup (4, \infty)$

There are several different results that could result from an "OR" statement. The graphs could be pointing in different directions with no overlap (like in the example above), pointing in the same direction, or pointing in opposite directions with an overlap. In the table below, notice how interval notation works for each of these cases.

"OR" Inequalities

Type	Example	Directions
Arrows pointing in opposite directions with NO overlap	 Interval Notation: $(-\infty, -2] \cup (4, \infty)$	In this graph, both graphs can be true for the inequality.

Arrows point in same directions	 Interval Notation: $(-\infty, 1)$	As the graphs overlap, we take the largest graph for our solution.
Arrows point in opposite directions and do overlap	 Interval Notation: $(-\infty, \infty)$ or $\mathbb{R}$	When the graphs are combined, they cover the entire number line.

HINT

Notice in the last case, the numbers cover the entire number line. We can express this with the symbol $\mathbb{R}$, which is the set of all real numbers.

2. "AND" Inequalities

The second type of compound inequality is an "AND" inequality. "AND" inequalities require both statements to be true. If one is false, they both are false. When we graph these inequalities we can follow a similar process, first graph both inequalities above the number line, but this time only where they overlap will be drawn onto the number line for our final graph. When our solution is given in interval notation it will be expressed in a manner very similar to single inequalities (there is a symbol that can be used for "AND", the intersection, $\cap$, but we will not use it here).

EXAMPLE Suppose the value of x falls under $8 \geq 3x - 7$ and $5x > 3x + 4$. Solve each inequality, graph the solution, and give interval notation of the solution.

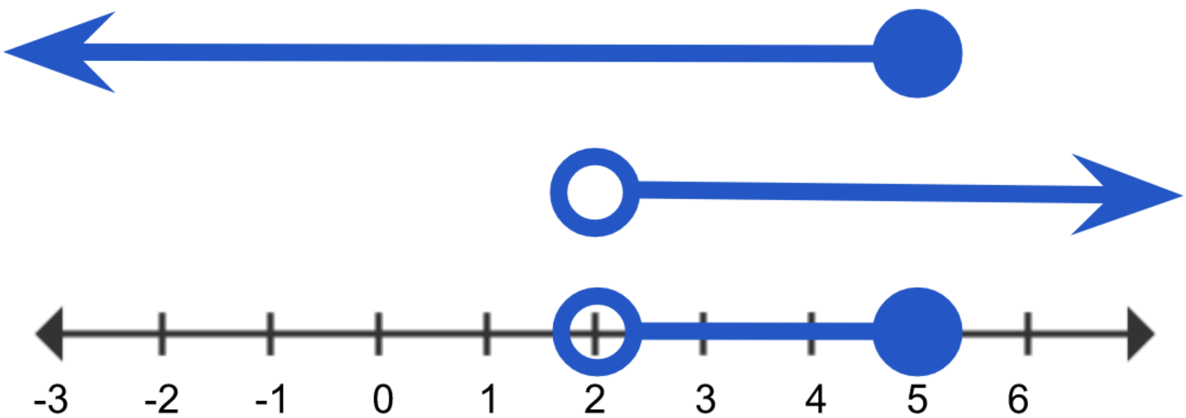
$8 \geq 3x - 7$ $+7 \quad\quad +7$	Solve by first adding 7 to both sides.
$\dfrac{15}{3} \geq \dfrac{3x}{3}$	Divide both sides by 3
$5 \geq x$	Rewrite with x on the left side
$x \leq 5$	Our solution

$$\begin{array}{r} 5x > 3x + 4 \\ \underline{-3x \quad -3x} \end{array}$$ Solve by first subtracting $3x$ from both sides.

$$\frac{2x}{2} > \frac{4}{2}$$ Divide both sides by 2

$$x > 2$$ Our solution

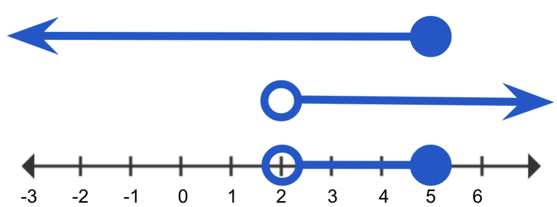
Graph the inequalities separately above a number line and then combine for a final graph.

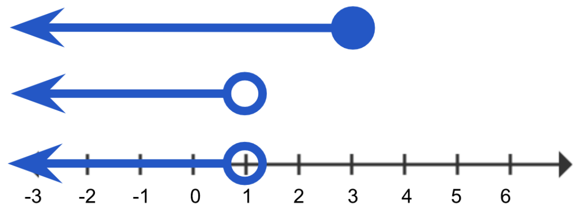
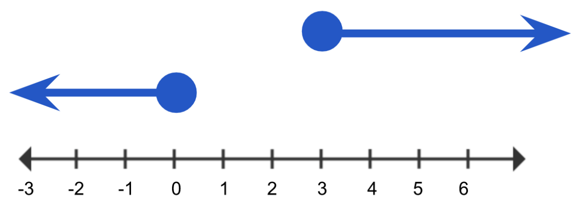


Interval notation: $(2, 5]$

Again, as we graph "AND" inequalities, only the overlapping parts of the individual graphs makes it to the final number line. As we graph "AND" inequalities, there are three different types of results we could get. The first is from in the above example.

"AND" Inequalities

Type	Example	Directions
Arrows pointing in opposite directions and overlap	 Interval Notation: $(2, 5]$	In this graph, only the overlapping parts of the individual graphs makes it to the final number line.

Arrows point in same directions	 Interval Notation: $(-\infty, 1)$	In this graph, the overlap is only the smaller graph, so this is what makes it to the final number line.
Arrows point in opposite directions and do NOT overlap	 Interval Notation: No Solution or $\varnothing$	In this graph, there is no overlap of the parts. Because of this, no value makes it to the final number line

HINT

Notice how interval notation is expressed in each case. In the last case, there was no solution, so we could also use the symbol ∅.

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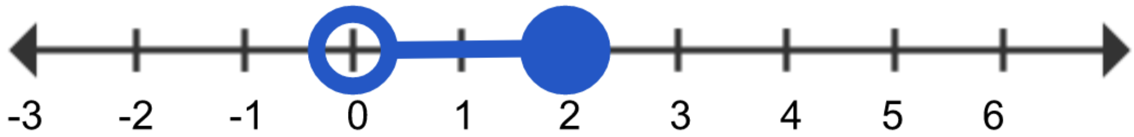
3. Another "AND" Inequality

The third type of compound inequality is a special type of "AND" inequality. When our variable (or expression containing the variable) is between two numbers, we can write it as a single math sentence with three parts, such as 5 less than *x* less or equal than 8, to show *x* is between 5 and 8 (or equal to 8). When solving these type of inequalities, because there are three parts to work with, to stay balanced we will do the same thing to all three parts (rather than just both sides) to isolate the variable in the middle. The graph then is simply the values between the numbers with appropriate brackets on the ends.

EXAMPLE Suppose the value of *x* falls under $-6 \leq -4x + 2 < 2$. Solve the inequality, graph the solution, and give interval notation of the solution.

$\begin{array}{rcl} -6 \leq & -4x + 2 & < 2 \\ \underline{-2} & \underline{-2} & \underline{-2} \end{array}$	Subtract 2 from all three parts
$\dfrac{-8}{-4} \leq \dfrac{-4x}{-4} < \dfrac{0}{-4}$	Divide all three parts by -4. Don't forget to flip the symbols!
$2 \geq x > 0$	Flip entire statement so values get larger left to right
$0 < x \leq 2$	Our solution

Graph the inequality on the number line.



Interval notation: $(0, 2]$

SUMMARY

Compound inequalities have two or more inequalities. The solution set has to satisfy all of the inequalities. **"OR" inequalities** also have two or more inequalities, but the solution set satisfies any but not necessarily all of the inequalities. With **"AND" inequalities**, only the overlapping parts of the individual graphs makes it to the final number line. When you're solving **another "AND" inequality** with an algebraic expression between two inequality symbols, any operation done between the symbols must also be done on the other side of both inequality symbols.