Conditional Probability

by Sophia

WHAT'S COVERED

This tutorial will cover the topic of conditional probability.

1. Conditional Probability

What's the probability of something happening if something else has already happened? This type of probability is called **conditional probability**.

If you are trying to determine the probability that event B will occur, given that another event (A) has already occurred, it is written with the following notation:

$$P(B, \text{given } A) = P(B \mid A)$$

This is the notation of the probability that event B occurs, given event A has also occurred -- the vertical bar is pronounced "given".

Conditional probability is calculated with the following formula:

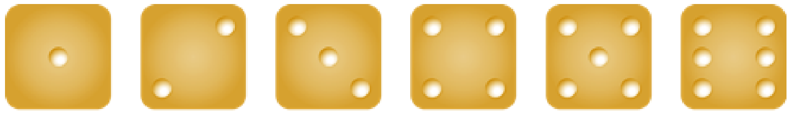
FORMULA TO KNOW

Conditional Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

The probability that event B occurs, given that event A also occurs, is the probability of both A and B occurring divided by the probability of event A.

EXAMPLE Suppose you are rolling a standard six-sided die. What is the probability of rolling a 4?



This question is not a conditional probability yet because the question is simply about the probability that the number is a 4. To find this answer, simply find the probability of rolling a 4.

$$P(4) = \frac{1}{6}$$

Of the six possible outcomes, only one outcome would result in rolling a 4.

Suppose you are told that the number you rolled was even. What is the probability of getting a 4, given this information?

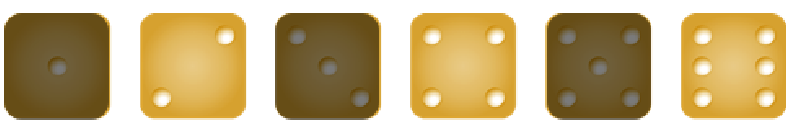
We can use the above formula that tells us that we need to divide the probability of rolling both a 4 and an even number by the probability of getting an even number. The probability of the event of getting a 4 and an even number is just 1 out of 6; there is only one die that has both characteristics. The probability of getting an even number is 3 out of 6.

$$P(4 \mid Even) = \frac{P(4 \text{ and } Even)}{P(Even)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

HINT

Notice that since the fractions have the same denominator of 6, we can simplify the fraction by canceling the 6 and writing 1 over 3.

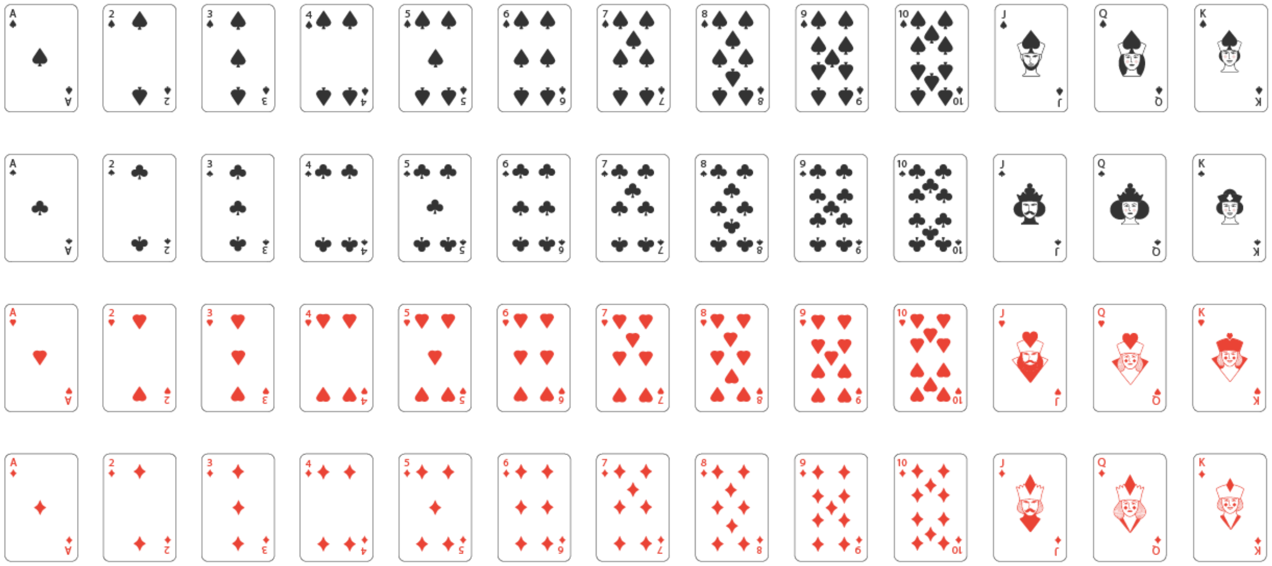
This makes sense if you consider the information we were given. If we knew that die was going to be even, this narrows our choices down to only 2, 4, or 6. Of those three outcomes, only one is a 4.



EXAMPLE In a well-shuffled, standard 52 card deck, what is the probability that a card is a face card, given that it is NOT a red card?

HINT

When dealing with cards, remember that a face card is a Jack, Queen, or King.



The probability that a card is a face card given that it is NOT a red card can be expressed as the conditional probability:

$$P(Face\ Card|Not\ Red) = \frac{P(Face\ Card\ and\ Not\ Red)}{P(Not\ Red)}$$

The probability that a card is both a face card AND not a red card is 6 over 52, as there are only 12 face cards in a deck, 6 that are red and 6 that are black (or "not red").

The probability that a card is NOT a red card is $\frac{26}{52}$ as half of the deck of cards are black (or "not red").

So the probability of a card being a face card given that it is a NOT a red card is:

$$P(Face\ Card|Not\ Red) = \frac{P(Face\ Card\ and\ Not\ Red)}{P(Not\ Red)} = \frac{\frac{6}{52}}{\frac{26}{52}} = \frac{6}{26} \approx 0.23\ or\ 23\%$$

TRY IT

At the local college, 70% of classes have final exams and 40% have final research papers. 20% have both research papers and final exams.

Calculate the probability a course will have a research paper, given that it has a final exam. +

Apply the conditional probability formulas as follows:

$$P(Paper \mid Exam) = \frac{P(Exam\ and\ Paper)}{P(Exam)}$$

$$P(Paper \mid Exam) = \frac{0.20}{0.70} = 0.286\ or\ about\ 29\%$$

TERM TO KNOW

Conditional Probability

The probability that one event occurs, given that another event has already occurred.

SUMMARY

Conditional probability is the probability of some second event occurring, given that some first event has already occurred. It's calculated by dividing the joint probability of the two events by the probability of the existing event (the one that's already happening). This formula works for all events. This isn't a special formula that works only for independent events or only for mutually exclusive events.

TERMS TO KNOW

Conditional Probability

The probability that one event occurs, given that another event has already occurred.

FORMULAS TO KNOW

Conditional Probability

$$P(B \mid A) = \frac{P(A\ and\ B)}{P(A)}$$