Continuously Compounding Interest

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WHAT'S COVERED

In this lesson, you will learn how to solve for account balance when interest is compounded continuously. Specifically, this lesson will cover:

1. Compound Interest

Bank and investment accounts appreciate in value according to compounding interest. For many, the account has an annual percentage rate (APR) that is compounded periodically throughout the year. This means that a portion of the APR is applied each time the interest is compounded.

EXAMPLE If interest is compounded quarterly, one-fourth of the interest is applied each quarter, for a total of four times per year.

We can represent compounding interest using the following formula:

FORMULA TO KNOW

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

In this formula,

- A is the account balance after t number of years.
- P is the principal (initial starting balance).
- r is the annual percentage rate (expressed as a decimal).
- n is the number of times per year interest is compounded.
- t is time in years.

2. Continuously Compounding Interest

When interest is compounding continuously, we can think of the variable n (the number of times per year interest is compounded) as being infinitely large. How do we make sense of our formula if n is an infinitely large number? Let's isolate the part of our formula that relies on the variable n, and consider when n approaches infinity:

$$\lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^{nt}$$

HINT

For these purposes, it isn't so important that we understand this notation, but this denotes the limit of the expression as n approaches infinity. The limit represents a value that the expression approaches, but will not exceed.

As it turns out, as n gets infinitely larger, the expression $\left(1 + \frac{r}{n}\right)^{nt}$ simplifies to e^{rt}. Recall that e is a mathematical constant, approximately equal to 2.718282. Having this in mind, we can adjust our interest formula for **continuously compounding interest**:

FORMULA TO KNOW

Continuously Compounding Interest

$$A = Pe^{rt}$$

In this formula,

- A is the account balance after t number of years
- P is the principal (initial starting balance)
- e is the mathematical constant, approximately equal to 2.718282
- r is the annual percentage rate (expressed as a decimal)
- t is time in years

3. Solving for Account Balance

Now that we have a formula to use when interest is compounded continuously, let's use it to solve some problems with account balances.

EXAMPLE An account has an initial balance of $1150.00 and has an APR of 2%, which is compounded continuously. What is the balance of the account after 3 years, assuming no additional deposits or withdrawals are made?

Let's identify values we can plug in for the variables in our formula:

- $P = 1150$
- $r = 0.02$
- $t = 3$

We can plug these into the variables in our continuously compounded interest formula:

$A = Pe^{rt}$	Plug in $P = 1150,\ r = 0.02,\ t = 3$
$A = 1150e^{0.02(3)}$	Evaluate multiplication in exponent
$A = 1150e^{0.06}$	Apply exponent to *e*
$A = 1150(1.061837)$	Multiply by principal balance
$A = 1221.11$	Our solution

The account balance will be $1,221.11 after 3 years.

HINT

If your calculator has the *e* button, use it to get the most accurate answer. If not, use the approximation 2.718282 and use as many decimal digits as possible during your calculations, and round to the nearest cent at the very end only. If you round too often during the calculations, you may get the same dollar amount, but the cents will likely be off.

4. Solving for Growth Time

We can also use the formula for continuously compounding interest to solve for the time it takes for the account to reach a certain value.

EXAMPLE A savings account has a balance of $4,500. The interest rate of the account is 3.5% annually, which is compounded continuously. How long will take for the account to reach a value of $7,000, assuming no additional deposits or withdrawals are made?

Again, let's match up given information with variables in our formula:

- $P = 4500$
- $r = 0.035$
- $A = 7000$

Here, we want to solve for *t*, so this remains our unknown. The equation for this situation is:

$A = Pe^{rt}$ Plug in $P = 4500$, $r = 0.035$, $A = 7000$

$7000 = 4500e^{0.035t}$ Solve for t

How can we solve for the variable t? One thing to note right away is that the variable t is an exponent here. The exponent $0.035t$ is applied *only* to the mathematical constant *e*, not the principal balance, *P*. So our first step should always be to divide both sides of the equation by the principal.

$7000 = 4500e^{0.035t}$ Divide by 4500

$1.556 = e^{0.035t}$ Solve for t

Now we are ready to undo the exponent in order to isolate *t*. This requires applying a logarithm to undo the exponent. The important thing here is to consider the base of the log. Since the base of the exponential is *e*, we should use the natural log to solve for *t*.

$1.556 = e^{0.035t}$ Take the natural log of both sides

$ln(1.5556) = ln\left(e^{0.035t}\right)$ Appy the Power Property of Logs

$ln(1.5556) = 0.035t \cdot ln(e)$ Simplify the left side with $ln(e) = 1$

$ln(1.5556) = 0.035t$ Divide both sides by 0.035

$\frac{ln(1.5556)}{0.035} = t$ Evaluate fraction

$12.62 = t$ Our solution

$12.62\ years$ Our solution

It will take 12.62 years for the account to reach a value of $7,000.

HINT

Notice the relationship between *e* and the natural log, ln. The natural log is the inverse of *e*. So when you take the natural log of *e*, or $\ln(e)$, this is simply equal to 1.

SUMMARY

Bank and investment accounts appreciate in value according to **compound interest**. Some bank accounts gain interest continuously. With **continuously compounding interest**, *n*, the number of times interest is compounded per year, is an infinitely large number. The continuously compounded interest formula uses the mathematical constant e, which is equal to approximately 2.718281. When **solving for account balance**, you will solve for A. For **solving for growth time**, you will solve for t using logarithms.

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