

Converting Between Forms

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WHAT'S COVERED

In this lesson, you will learn how to convert a linear equation from standard form to slope-intercept form and vice versa. Specifically, this lesson will cover:

1. Why and When to Convert
2. Convert from Standard Form to Slope-Intercept Form
3. Convert from Point-Slope Form to Slope-Intercept Form
4. Convert from Slope-Intercept form to Standard Form

1. Why and When to Convert

Linear equations can be written in several different forms. Sometimes, it is beneficial to have the equation to the same line written in a different form, so that you can more readily draw certain information about the line just by looking at its equation. For instance, having an equation written in standard form doesn't make it easy to identify the line's slope, or at least not as easy as the same line written in slope-intercept form or point-slope form. Likewise, if an equation is written in slope-point form, and you wish to easily identify the line's y-intercept, converting the equation into the slope-intercept form will be helpful.

In this tutorial, we are going to go through a few examples of converting from one form to another, in order to more easily draw conclusions about a line's slope, intercept, or point on the line. Here are the forms again as a review:



FORMULA TO KNOW

Slope-Intercept Form of a Line

$$y = mx + b$$

Point-Slope Form of a Line

$$y - y_1 = m(x - x_1)$$

Standard Form of a Line

$$Ax + By = C$$

2. Convert from Standard Form to Slope-Intercept Form

When you need to find the slope and intercept of an equation, it is best to convert the equation into slope-intercept form (if it is not already in that form).

⇒ **EXAMPLE** Find the slope and intercept for the equation $2x - 3y = 15$.

If we wish to identify the line's slope and intercept, it would be wise for us to convert this equation into slope-intercept form, $y = mx + b$, so we can simply look at m and b for that information. As you read the steps below, keep in mind that the overall goal is to get the y -term by itself on one side of the equation, and then cancel the coefficient in front of y .

$2x - 3y = 15$ This equation is currently in standard form. Convert to slope-intercept form by first subtracting $2x$ from both sides

$-3y = 15 - 2x$ Divide all terms by -3 (note the x -term coefficient will now be positive)

$y = -5 + \frac{2}{3}x$ Rearrange to $y = mx + b$

$y = \frac{2}{3}x - 5$ Slope-Intercept Form

In this form, it is clear that the line has a slope of two-thirds and a y -intercept at $(0, -5)$.



BIG IDEA

When converting from standard form to slope-intercept form, isolate the y -term to one side of the equation, and then divide by its coefficient. This will leave y alone on one side of the equation. Then, just rearrange the terms so that mx is first, and b follows.

3. Convert from Point-Slope Form to Slope-Intercept Form

While both point-slope form and slope-intercept form provide information about a line's slope, and technically a point on the line as well, point-slope form can give the location to any point on the line, whereas slope-intercept form gives only the y -coordinate to the y -intercept.

⇒ **EXAMPLE** Suppose a line has a slope of 4 and passes through the point $(-3, 7)$. What is the line's y -intercept?

Since we are given the slope and a point, we can first write the equation in point-slope form. Then we'll need to convert to slope-intercept form to find the y-intercept:

$$y - y_1 = m(x - x_1) \quad \text{This is the point-slope form equation. Substitute the known values: } m = 4, (x_1, y_1) = (-3, 7)$$

$$y - 7 = 4(x + 3) \quad \text{Distribute 4 into } (x + 3)$$

$$y - 7 = 4x + 12 \quad \text{Add 7 to both sides}$$

$$y = 4x + 19 \quad \text{Use this to find the y-intercept}$$

$$(0, 19) \quad \text{y-intercept}$$



BIG IDEA

To convert from point-slope form into slope-intercept form, distribute the slope, m , into the expression in parentheses. Then move the constant term attached to y to the other side of the equation. Finally, combine like terms to arrive at the equation in slope-intercept form.

4. Convert from Slope-Intercept form to Standard Form

Having an equation in standard form can help us easily calculate both x- and y-intercepts. This is because for each intercept, either x or y will be zero, making the entire x or y term in the equation equal to zero. In this final example, we are going to convert an equation from slope-intercept form into standard form, and then identify both x- and y-intercepts using the equation in standard form:

⇒ **EXAMPLE** Convert the equation $y = 2x + 3$ into standard form and identify the x-intercept and y-intercept.

$$y = 2x + 3 \quad \text{This equation is currently in slope-intercept form. Convert to standard form by first moving the x-term to the same side as the y-term. Subtract } 2x \text{ from both sides}$$

$$-2x + y = 3 \quad \text{Multiply everything by } -1, \text{ since the coefficient of } x \text{ should not be negative}$$

$$2x - y = -3 \quad \text{Standard form}$$

To find the x-intercept, substitute 0 in for y :

$$\begin{aligned}
 2x - y &= -3 \\
 2x - 0 &= -3 \\
 2x &= -3 \\
 x &= \frac{-3}{2}
 \end{aligned}$$

The x-intercept is at the point $\left(-\frac{3}{2}, 0\right)$.

To find the y-intercept, substitute 0 in for x:

$$\begin{aligned}
 2x - y &= -3 \\
 2 \cdot 0 - y &= -3 \\
 -y &= -3 \\
 y &= 3
 \end{aligned}$$

The y-intercept is at the point (0, 3).



HINT

Remember to multiply the entire equation by -1 if you have an x-term with a negative coefficient, like in the example above. We went from $-2x + y = 3$ to $2x - y = -3$. This is because we prefer the x-term to have a positive coefficient, so in these cases, the signs of all terms must switch.



BIG IDEA

To convert into standard form, the goal is to get the x-term and y-term on the same side of the equation, and the constant term on the other side. Remember that standard convention calls for the x-term to come first, and then the y-term. Additionally, the x-term should not have a negative coefficient.



SUMMARY

When understanding **why and when to convert**, it is beneficial to have the equation to the same line written in different forms so that you can more readily draw certain information about the line just by looking at its equation. The slope and y-intercept can be determined easily by looking at the equation of a line written in slope intercept form. When **convert from standard form to slope-intercept form** or **convert from point-slope form to slope-intercept form** to easily identify the slope and y-intercept. The x- and y-intercepts can be found easily when we **convert from slope-intercept form to standard form** and solve the equation of a line written in standard form for x and for y.

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FORMULAS TO KNOW

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$$y - y_1 = m(x - x_1)$$

Slope-Intercept Form of a Line

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Standard Form of a Line

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