Distance, Rate, and Time in a System of Equations

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WHAT'S COVERED

In this lesson, you will learn how to solve a distance, rate, and time problem using a system of equations. Specifically, this lesson will cover:

1. Distance, Rate, and Time Relationship

A speed limit sign, something we see every day, provides great insight into the relationship between distance (d), rate (r), and time (t). Take for instance a speed limit of 40 miles per hour. Speed is a rate: in this case, it's the ratio between miles and hours. Miles is a unit of distance, and hours is a unit of time. Therefore, we can say that rate equals distance divided by time.

We can use this relationship to write a few equivalent equations:

FORMULA TO KNOW

Distance, Rate, and Time

$$d = r \cdot t$$

$$r = \frac{d}{t}$$

$$t = \frac{d}{r}$$

In this lesson, we will work primarily with distance = rate • time or $d = r \cdot t$.

2. Combined Rates

In some situations, more than one rate has an effect on the situation, which in turn affects how we represent the situation mathematically. For example, imagine swimming across a river or lake. In one direction, the current of the water might work in your favor, making you faster, or at least making it easier for you to keep a certain pace. In the other direction, the current of the water might be working against you, making you slower, or making it harder for you to keep a certain pace.

When two rates work with each other (such as swimming with the current), we can add the two rates together. When rates work against each other (such as swimming against the current), the opposing rate is negative, leading to an expression involving subtraction.

- Rates working together: $r_1 + r_2$
- Rates working against each other: $r_1 - r_2$

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3. Solving a Distance, Rate, and Time Problem using a System of Equations

Similar to the mixture problems, when solving a distance, rate, time problem, you need to:

- define variables
- identify the system of equations

EXAMPLE A swimmer can swim a half-mile with the current in 6 minutes. However, traveling back against the current, the same distance takes 15 minutes. Assuming the swimmer travels at a constant speed during both trips, what is the speed of the swimmer, and what is the speed of the current?

To solve this problem, we will set up two equations to be part of our system. Both equations will be forms of $d = r \cdot t$, and we'll use combined rates for the variable *r*. A few things are given to us in our equation: the distance in both equations is 0.5 miles. For the equation with rates working together, the time is 6 minutes, and in the equation with rates working against each other, the time is 15 minutes. Since we know the distance and time, we need to solve for the rates, *r*.

Before we set up our equations, it is important that we have uniform units of measure. Because we typically express speed in miles per hour, we want to convert 6 minutes and 15 minutes into hours. To do this, we will divide each number by 60 (because there are 60 minutes in 1 hour). 6 minutes becomes 0.1 hours, and 15 minutes becomes 0.25 hours.

It is also important that we establish our variables:

$$r_1 = \text{rate of the swimmer}$$
$$r_2 = \text{rate of the current}$$

Using $d = r \cdot t$, we can develop two equations in our system. One equation represents swimming with the current, and one equation represents swimming against the current.

$$0.5 = (r_1 + r_2)(0.1)$$
$$0.5 = (r_1 - r_2)(0.25)$$

The first equation tells us that it took the swimmer 0.1 hours (6 minutes) to swim 0.5 miles when he was swimming with the current. The second equation tells us that it took the swimmer 0.25 hours (15 minutes) to swim 0.5 miles when he was swimming against the current.

Now that we have a system set up, our job is to solve for the rate of the swimmer and the rate of the current. To do this, we can create equivalent equations by dividing each equation above by its respective time. In the first equation, divide 0.5 by 0.1 to get $5 = r_1 + r_2$. In the second equation, divide 0.5 by 0.25 to get $2 = r_1 - r_2$. Our simplified system is now:

$$5 = r_1 + r_2$$
$$2 = r_1 - r_2$$

To solve for our rates, we notice that if we add the two equations, one of the terms will cancel, because it is positive in one equation, and negative in the other. When we add the equations, we can solve for one of the rates:

$5 = r_1 + r_2$ $2 = r_1 - r_2$	Using our system of equations, add the equations together and the term r_2 cancels
$7 = 2r_1$	Divide both sides by 2
$3.5 = r_1$	Rate of the swimmer

This solution tells us that the rate of the swimmer, r_1, is 3.5 mph.

Now that we know the value of one rate, we can substitute that value into either of our two original equations in the system to find the value of the other rate:

$5 = r_1 + r_2$	Using the first equation, substitute the 3.5 for r_1
$5 = 3.5 + r_2$	Subtract 3.5 from both sides
$1.5 = r_2$	Subtract 3.5 from both sides

This solution tells us that the rate of the current, r_2, is 1.5 mph.

SUMMARY

Recall that the **distance, rate, and time relationship** can be expressed with the equation distance equals rate multiplied by time. A system of equations can be used to represent two rates working with and against each other. When rates work together, the **combined rate** is the sum of the individual rates and when rates work against each other, the combined rate is the difference of the individual rates. When **solving a distance, rate, and time problem using a system of equations**, use the graphing, substitution, or addition method to solve the system.

FORMULAS TO KNOW

Distance, Rate, and Time

$d = r \cdot t$

$r = \frac{d}{t}$

$t = \frac{d}{r}$