# Divide Complex Numbers

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WHAT'S COVERED

In this lesson, you will learn how to divide two complex numbers. Specifically, this lesson will cover:

## 1. Complex Numbers

A complex number is a number in the form $a+bi$, containing both a real part and an imaginary part. The imaginary part is followed by *i*, which is the imaginary unit, $\sqrt{-1}$.

The process of dividing two complex numbers has many similarities with the process of rationalizing denominators. If you have studied rationalizing denominators, you may be familiar with using a conjugate in order to clear any irrational expressions from the denominator of a fraction. With complex number division, we use complex number conjugates to clear imaginary numbers from the denominator. Before we get to examples, let's review conjugates.

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## 2. Conjugate Review

Recall that a **conjugate** of a binomial is a binomial with the opposite sign between its terms. Finding the conjugate of a complex number is straightforward. We simply reverse the sign in between the real part and the imaginary part. When we see plus signs, we write minus signs, and vice versa. Here is a table with some complex numbers and their complex conjugates:

| Complex Number | Complex Conjugate |
|---|---|
| $8+4i$ | $8-4i$ |
| $-6+3i$ | $-6-3i$ |
| $7-5i$ | $7+5i$ |
| $-2-9i$ | $-2+9i$ |

HINT

Only the sign in front of the imaginary number changes. Notice that the sign in front of the real number stays the same when writing a complex number and its complex conjugate.

Next, we will see how complex conjugates help us solve division problems involving complex numbers.

 

TERM TO KNOW

**Conjugate**

The conjugate of a binomial is a binomial with the opposite sign between its terms.

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# 3. Dividing Complex Numbers

To set up a division problem with complex numbers, we want to write it as a fraction. For instance, to divide $4+7i$ by $3+2i$, we write:

$$\frac{4+7i}{3+2i}$$

Here is where the complex conjugate comes into play. In order to clear imaginary numbers from the denominator, we use the complex conjugate of the denominator to create a second fraction. In this fraction, the complex conjugate will make up the numerator and denominator. Since both the numerator and denominator are identical quantities, the fraction has a value of 1, and can be multiplied by our complex division problem and maintain its value.

BIG IDEA

Use the complex conjugate of the denominator to create a second fraction with a value of 1. In order to have a value of 1, the complex conjugate must make up both the numerator and denominator of this fraction. Multiplying the two fractions will not change the value, but it will allow us to simplify our solution.

By creating this fraction, we have really set up a fraction multiplication problem. We'll need to multiply across numerators, and then multiply across denominators. Let's first multiply across numerators and simplify the numerator of our solution as much as we can:

EXAMPLE Divide $4+7i$ by $3+2i$.

| | |
|---|---|
| $\frac{4+7i}{3+2i}$ | Multiply by a second fraction with the conjugate $3-2i$ in the numerator and denominator |
| $\frac{4+7i}{3+2i}\cdot\frac{3-2i}{3-2i}$ | Multiply the two fractions |
| $\frac{(4+7i)(3-2i)}{(3+2i)(3-2i)}$ | Use FOIL to evaluate numerator |
| $\frac{12-8i+21i-14i^2}{(3+2i)(3-2i)}$ | Combine like terms in numerator |
| $\frac{12+13i-14i^2}{(3+2i)(3-2i)}$ | Rewrite $-14i^2$ as $+14$ |
| $\frac{12+13i+14}{(3+2i)(3-2i)}$ | Combine like terms in numerator |
| $\frac{26+13i}{(3+2i)(3-2i)}$ | Evaluate denominator |

 

It is in the multiplication of denominators that we'll see why using the complex conjugate is so helpful. While working through this next set of multiplication, pay particular attention when we combine the two imaginary numbers, and when we simplify the squared imaginary unit.

| | |
|---|---|
| $\frac{26+13i}{(3+2i)(3-2i)}$ | Use FOIL to evaluate denominator |
| $\frac{26+13i}{9-6i+6i-4i^2}$ | Combine like terms in denominator |
| $\frac{26+13i}{9-4i^2}$ | Rewrite $-4i^2$ as $+4$ |
| $\frac{26+13i}{9+4}$ | Simplify denominator |
| $\frac{26+13i}{13}$ | Separate into two fractions |

By using the denominator's complex conjugate, the i-terms in the denominator will always cancel each other out. Additionally, the $i^2$ term simplifies to a real number. The result is that the denominator is a purely real number, with no imaginary components.

Now we can simplify our fraction to get our final solution:

| | |
|---|---|
| $\frac{26+13i}{13}$ | Separate into two fractions |
| $\frac{26}{13}+\frac{13i}{13}$ | Simplify fractions |
| $2+i$ | Our solution |

↪ EXAMPLE Divide $2+4i$ by $3-5i$.

| | |
|---|---|
| $\frac{2+4i}{3-5i}$ | Multiply by a second fraction with the conjugate $3+5i$ in the numerator and denominator |
| $\frac{2+4i}{3-5i}\cdot\frac{3+5i}{3+5i}$ | Multiply the two fractions |
| $\frac{(2+4i)(3+5i)}{(3-5i)(3+5i)}$ | Use FOIL to evaluate numerator and denominator |
| $\frac{6+10i+12i+20i^2}{9+15i-15i-25i^2}$ | Combine like terms in numerator and denominator |
| $\frac{6+22i+20i^2}{9-25i^2}$ | Rewrite $+20i^2$ as $-20$ and $-25i^2$ as $+25$ |
| $\frac{6+22i-20}{9+25}$ | Combine like terms in numerator and denominator |
| $\frac{-14+22i}{34}$ | Separate into two fractions |

$\frac{-14}{34}+\frac{22i}{34}$ Simplify fractions

$-\frac{7}{17}+\frac{11}{17}i$ Our solution

## SUMMARY

**Complex numbers** consist of a real part and an imaginary part. The square root of negative 1 is imaginary, because no real number squared results in a negative number. **Conjugates** are used when **dividing complex numbers** to eliminate a complex number from the denominator. Simplify expressions with an *i* squared term by substituting negative 1 for *i* squared, multiplying by the coefficient, and writing as a real number with the opposite sign.



## TERMS TO KNOW

**Conjugate**
The conjugate of a binomial is a binomial with the opposite sign between its terms.