"Either/Or" Probability

by Sophia

WHAT'S COVERED

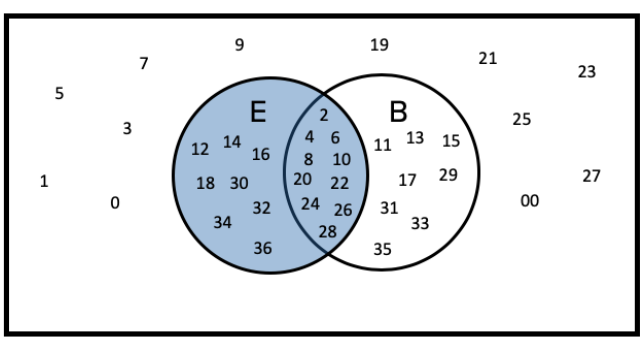
In this tutorial, you're going to learn about “or” probability for overlapping events. Our discussion breaks down as follows:

1. Either/Or Probability for Overlapping Events

Either/or probability for overlapping events refers to the probability of either A, or B, or both, occurring.

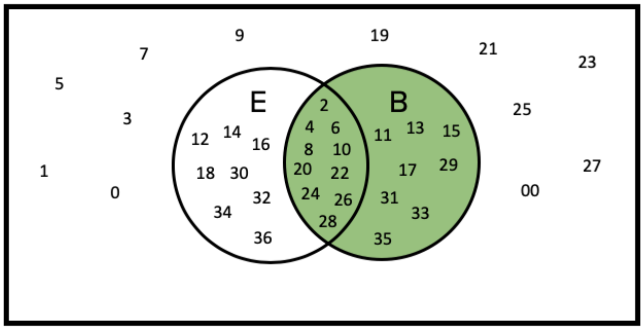
EXAMPLE Consider the roulette wheel

What's the probability of getting an even number on a roulette wheel? This is not an “or” probability. We're simply finding the probability of spinning an even number. Looking just in the even circle, 18 numbers are even on the roulette wheel, out of 38 sectors. Zero and double zero don't count as even.



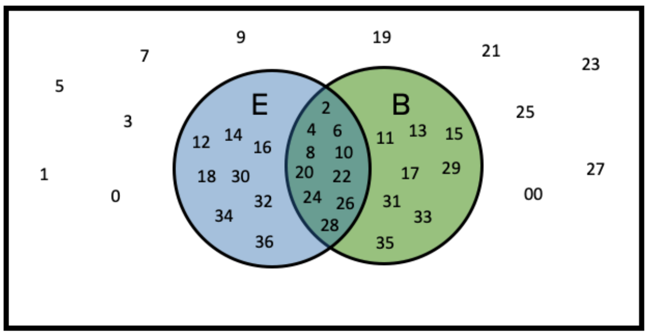
$$P(Even) = \frac{18}{38} = 0.47$$

What is the probability of black on the roulette wheel? It's the same idea. Look in the circle that represents black numbers, it's also 18 out of 38, coincidentally the same number.



$$P(Black) = \frac{18}{38} = 0.47$$

What's the probability of even *or* black? Count up the numbers that are in either even or black. You get 26 out of the 38.



$$P(Even\ or\ Black) = \frac{26}{38} = 0.68$$

Now the question is, why is the probability of E or B not equal to the probability of E plus probability of B?

$$\frac{26}{38} \neq \frac{18}{38} + \frac{18}{38}$$
$$P(Even\ or\ Black) \neq P(E) + P(B)$$

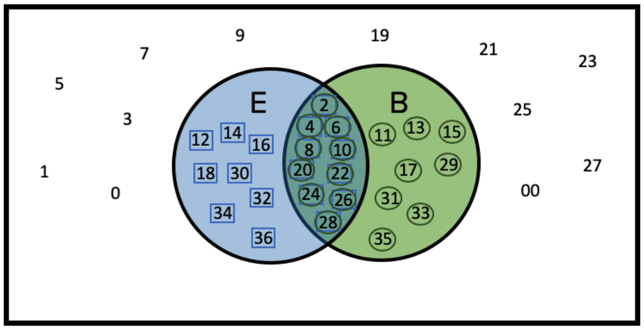
When you add the probability of an even number (18/38) plus the probability of a black number (18/38), might might expect to get 36/38, but you don't--you get 26/38. The reason for the miscount is because some of the numbers were counted twice. Some were counted as even, and some were counted again as being black, whereas we should only have counted them once.

Which values were double counted? It was the numbers that were black *and* even. You make this adjustment by first adding P(black) to P(even), then subtract out the things that got double counted P(black & even) so they are only counted once. The double counts were the numbers that were both black and even, represented in the overlap of the Venn diagram.

$P(E \text{ or } B) = P(E) + P(B) - P(E \text{ and } B)$

This represents the **either/or probability for overlapping events**: the probability of E or B is equal to the probability of E plus the probability of B, minus the probability of E and B.

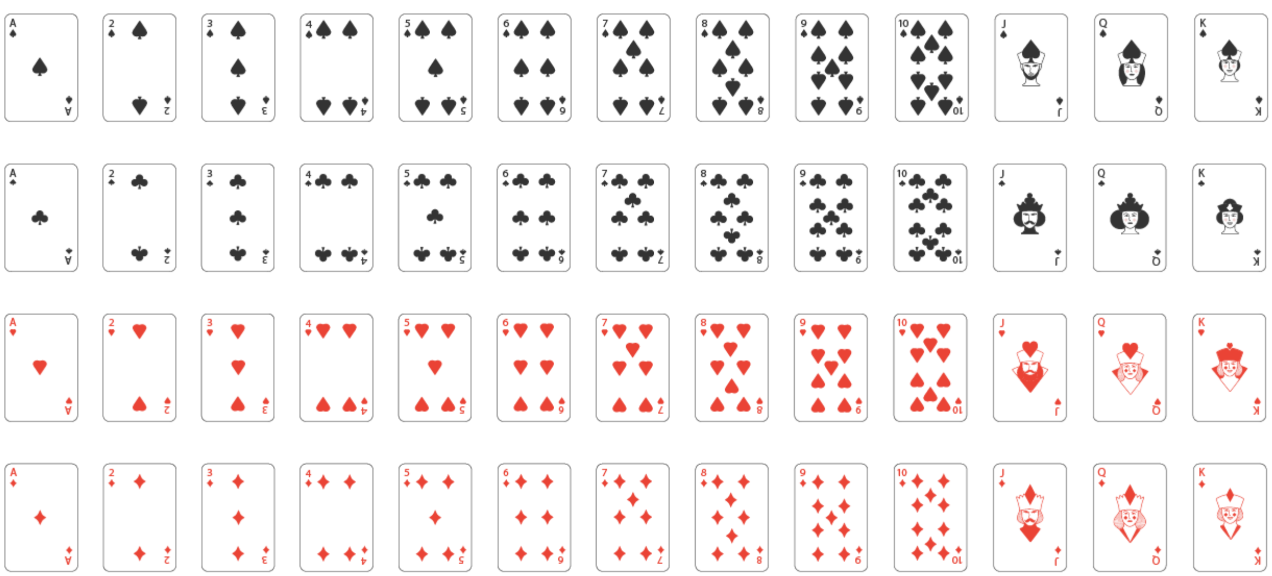
Let's illustrate this concept by boxing the probability of all even numbers and circling the probability of B numbers.



Notice how the numbers in the middle got both circled and boxed. You only want to count them once. But when we include the numbers with the E's and also with the B's, they were counted twice. Remove one of those markings to only count everything once.

$$P(\text{Even or Black}) = P(\text{Even}) + P(\text{Black}) - P(\text{Even and Black})$$
$$P(\text{Even or Black}) = \frac{18}{38} + \frac{18}{38} - \frac{10}{38}$$
$$P(\text{Even or Black}) = \frac{26}{38}$$

TRY IT



In a standard deck of cards, what is the probability of drawing a heart or a queen? +

Apply the formula or either/or probability for overlapping events.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(\text{Heart or Queen}) = P(\text{Heart}) + P(\text{Queen}) - P(\text{Heart and Queen})$

$P(\text{Heart or Queen}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} \text{ or } 31\%$

FORMULA TO KNOW

Either/Or Probability for Overlapping Events

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

TERM TO KNOW

Either/Or Probability for Overlapping Events

The probability that either of two events occurs is equal to the sum of the probabilities of the two events, minus the joint probability of the two events happening together. Also known as the "General Addition Rule".

2. General Addition Rule for Non-Overlapping Events

It’s important to note, this formula also works for non-overlapping events. Since mutually exclusive events, A and B, can't happen at the same time, the probability that both A and B occur is zero. It's impossible for A and B to both happen.

$\text{For Non-Overlapping Events:}$
$P(A \text{ and } B) = 0$

Therefore, when we factor that into our formula, we end up with the probability of A plus the probability of B minus 0. When it simplifies down, you may recognize this. This was the special addition rule for non-overlapping events. What we see, then, is that the special addition rule for non-overlapping events is a special case of the general addition rule.

$\text{For Non-Overlapping Events:}$
$P(A \text{ or } B) = P(A) + P(B) - \cancel{P(A \text{ and } B)}$
$P(A \text{ or } B) = P(A) + P(B)$

TRY IT

In a standard deck of cards, what is the probability of drawing a face card (J, Q or K) or a 10? +

Apply the formula for either/or probability for non-overlapping events. $P(A \text{ or } B) = P(A) + P(B)$

$$P(\text{Face Card or } 10) = P(\text{Face Card}) + P(10)$$

$$P(\text{Face Card or } 10) = \frac{12}{52} + \frac{4}{52} = \frac{16}{52} \text{ or } 31\%$$

SUMMARY

Either/or probability for overlapping events is calculated by adding the probabilities of the two events, then subtracting their joint probability. The reason you subtract out the joint probability is that it is counted in both of their individual probabilities. You don't want to count it twice; you only want it counted once. With mutually exclusive (non-overlapping) events, the joint probability is zero, and it simplifies down to the special addition rule, which is a special case of the general addition rule.

Good luck!

TERMS TO KNOW

Either/Or Probability for Overlapping Events

The probability that either of two events occurs is equal to the sum of the probabilities of the two events, minus the joint probability of the two events happening together. Also known as the "General Addition Rule".

FORMULAS TO KNOW

Either/Or Probability for Overlapping Events

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$