Evaluating Radicals

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WHAT'S COVERED

In this lesson, you will learn how to evaluate a given radical. Specifically, this lesson will cover:

1. Square Roots

Square roots are the most common type of radical used. A square root "unsquares" a number. For example, because $5^2 = 25$ we say the square root of 25 is 5. The square root of 25 is written as $\sqrt{25}$.

EXAMPLE

Square Root	Square Powers
$\sqrt{1} = 1$	$1^2 = 1$
$\sqrt{4} = 2$	$2^2 = 4$
$\sqrt{9} = 3$	$3^2 = 9$
$\sqrt{121} = 11$	$11^2 = 121$
$\sqrt{625} = 25$	$25^2 = 625$
$\sqrt{-81} = \text{undefined}$	Not possible

HINT

In the last example, $\sqrt{-81}$ is undefined, as negatives have no square root. This is because if we square a positive or a negative, the answer will be positive (or zero). Thus we can only take square roots of non-negative numbers. In another lesson, we define a method we can use to work with and evaluate negative square roots, but for now, we will simply say they are undefined.

Not all numbers have a nice even square root. For example, if we found $\sqrt{8}$ on our calculator, the answer would be 2.8284271247461900976033774484 19... and even this number is a rounded approximation of the square root. Decimal approximations will work in most cases, but you may need the exact value, in which case you will express using the radical symbol, rather than expressing as a decimal.

HINT

When evaluating square roots, look for perfect squares. Perfect square are numbers such as 2, 4, 9, 16 (they are integers squared). You can also use your calculator's radical button, $\sqrt{\ }$, or apply a $\frac{1}{2}$ exponent.

2. Cube Roots

Just like square roots undo squaring a number, cube roots undo cubing a number. For example, because $2^3 = 8$, we say that the cube root of 8 is 2. The cube root of 8 is written as $\sqrt[3]{8}$ with a 3 as the index of the radical, to indicate a cube root.

EXAMPLE

$\sqrt[3]{27} = 3$

$\sqrt[3]{64} = 4$

$\sqrt[3]{-8} = -2$

Notice that the cube root of a negative number is results in a real number. This is because a negative number cubed is a negative number. So while square roots of negative numbers are non-real numbers, cube roots of negative numbers are real numbers. This actually holds true for all even roots and odd roots.

BIG IDEA

Taking an even root of a negative number leads to a non-real solution because any negative number raised to an even power is positive. However, taking an odd root of a negative number leads to a real number solution, because raising a negative number to an odd power results in a negative number.

HINT

When evaluating a cube root, look for perfect cubes. These will result in an integer, because perfect cubes are integers cubed. Some examples of perfect cubes are: 1, 8, 27, 64, and 125. You can also use your calculator's cube root button, $\sqrt[3]{\ }$, or raise to the fractional power $\frac{1}{3}$.

3. Higher Roots

While square and cube roots are the most common type of radical we work with, we can take higher roots of numbers as well: cube roots, fourth roots, fifth roots, etc. Consider this **definition of radicals**:

FORMULA TO KNOW

Definition of Radicals

$\sqrt[m]{a} = b \text{ if } b^m = a$

The small letter *m* inside the radical is called the index. It tells us which root we are taking, or which power we are “un-doing”. For square roots the index is 2. As this is the most common root, the two is not usually written.

DID YOU KNOW

The word for root comes from the French mathematician Franciscus Vieta in the late 16th century.

Take a look at several higher roots:

EXAMPLE

Higher Roots	Higher Powers
$\sqrt[3]{125} = 5$	$5^3 = 5 \cdot 5 \cdot 5 = 125$
$\sqrt[4]{81} = 3$	$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
$\sqrt[5]{32} = 2$	$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
$\sqrt[3]{-64} = -4$	$-4^3 = -4 \cdot -4 \cdot -4 = -64$
$\sqrt[3]{-27} = -3$	$-3^3 = -3 \cdot -3 \cdot -3 = -27$
$\sqrt[4]{-16} = \text{undefined}$	Not possible

This last example is not possible, or undefined, because if we take any positive or negative number to the fourth power, the answer will be positive (or zero). $3 \times 3 \times 3 \times 3 = 81$ or $(-2) \times (-2) \times (-2) \times (-2) = 16$. Thus we can only take the fourth roots of non-negative numbers.

HINT

We must be careful of a few things as we work with higher roots. First, its important not to forget to check the index on the root. $\sqrt{81} = 9$ but $\sqrt[4]{81} = 3$ This is because $9^2 = 81$ and $3^4 = 81$. Another thing to watch out for is negative value under roots. We can take an odd root of a negative number because a negative number raised to an odd power is still negative. However, we cannot take an even root of a negative number, because a negative number raised to an even power is positive.

SUMMARY

If you use a calculator to evaluate the radical, you first type in the radical button and then you type in the number. If your calculator does not have the necessary nth root button, you're going to use a fractional exponent and type it in as caret, open parentheses, one divided by *n*, close parentheses. And again, your *n* number is just the index of your radical. And finally, the nth root of a negative number will not evaluate to be a real number if *n* is even—like square root—but it will evaluate to be a real number if *n* is odd—like the cubed root.

FORMULAS TO KNOW

Definition of Radicals

$\sqrt[m]{a} = b \text{ if } b^m = a$