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Expected Return

by Sophia

WHAT'S COVERED

In this lesson, you will learn about the expected return on a single-period investment. Specifically, this lesson will cover:

1. Expected Return

Imagine that your friend offered you a chance to play his game of dice. You have to pay $1 to play and he keeps your money if you roll anything other than a 6. Would you play it? If you answered “it depends," you are ready to learn about expected value.

If your friend told you he would pay you $1 every time you roll a 6, you would be crazy to play. If your friend told you he would pay you $10 every time you roll a 6, you would be crazy not to play. Let's take a look at why:

$1 Payout						
Your Roll	⚀	⚁	⚂	⚃	⚄	⚅
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
Payout	-$1	-$1	-$1	-$1	-$1	+$1
Expected Value	$E(R)=\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(\$1)$ $E(R)=\left(\frac{5}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(\$1)=-\$\frac{5}{6}+\$\frac{1}{6}=-\$\frac{4}{6}=-\$0.67\text{ per bet}$					
BAD BET!						

$10 Payout						
Your Roll	⚀	⚁	⚂	⚃	⚄	⚅
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
Payout	-$1	-$1	-$1	-$1	-$1	+$10

Expected Value	$E(R)=\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(\$10)$ $E(R)=\left(\frac{5}{6}\right)(-\$1)+\left(\frac{1}{6}\right)(\$10)=-\$\frac{5}{6}+\$\frac{10}{6}=\$\frac{5}{6}=\$0.83 \text{ per bet}$
GOOD BET!	

Expected value, or **expected return**, is calculated by multiplying the probability that something will happen by the resulting outcome if it happens. In the two cases described above, there are five ways to lose and only one to win. But in Scenario 1, you can expect to lose $0.67 or 67% every time you bet, and in Scenario 2, you can expect to win $0.83 or 83% with each bet. This is a confusing topic in statistics and finance because on any given roll, there are only two outcomes — win or lose — and neither outcome involves $0.67 or $0.83 increments.

But to understand expected value, you have to imagine playing a particular game hundreds of times. If you were to sit at your friend's apartment and play the dice game 100 times, imagine what your bottom line would be. Even though you lose most of the time you roll, in the second scenario, when you win, you win big. After 100 games, you could expect to be up $83 ($0.83 per roll x 100 rolls). If you were foolish enough to play the game in the first scenario 100 times, you would expect to be down $67 to your friend.

BIG IDEA

In finance, evaluating your expected return is important, but never as simple as evaluating a game of dice.

IN CONTEXT

Imagine that you are going to buy a house a year from today and you have $20,000 saved for that investment. You can invest this money for a year and get some return on it. You are considering investing that money into stock of a ski/snowboard mountain resort in Colorado, so you go talk to your snowboarding friend who lives on your floor of the dorm. He tells you that any given winter could be super gnarly, totally chillax or wicked bogus, depending on how much it snows.

You start doing some research, and you realize that how the stock has performed has everything to do with how much it snows. You create three different categories based on snowfall and find the average stock return:

Winter Type	Annual Snowfall	Annual Stock Return
Super Gnarly (SG)	Over 20 feet	25%
Totally Chillax (TC)	Between 10 feet and 20 feet	10%
Wicked Bogus (WB)	Less than 10 feet of total snowdrop	-20%

Ok, so now what? Should you buy the stock?

If you said "it depends," that's a good answer. Being as smart as you are, you investigate recent weather patterns and you decide that there is a 25% chance of an SG year, a 60% chance of a TC year and only a 15% chance of a WB winter. You understand that past performance is never a guarantee of future results, but still you are happy with your research and you project an expected rate of return for your $20,000. How will it do?

Based on your research, you realize that the stock has an expected return of:

$$E(R) = (\text{Probability}_{SG}) \times (\text{Return}_{SG}) + (\text{Probability}_{TC}) \times (\text{Return}_{TC}) + (\text{Probability}_{WB}) \times (\text{Return}_{WB})$$
$$E(R) = (0.25)(0.25) + (0.60)(0.10) + (0.15)(-0.20)$$
$$E(R) = 0.0625 + 0.06 + -0.03 = 0.0925 = 9.25\%$$

If you were to invest the stock in the ski mountain, year after year, and your research proves accurate, you could expect to receive an average of 9.25% return each year. That is your expected return.

TERM TO KNOW

Expected Return
Computed by computing the product of the probability of a given event and the return in that case and adding together the products in each discrete scenario.

2. Variance

In probability theory and statistics, the **variance** is a measure of how far a set of numbers is spread out. It is one of several descriptors of a probability distribution, describing how far the numbers lie from the mean (expected value).

Understanding the concept of variance along with three typical asset classes — money market, bonds, and stocks — can help you build a portfolio for any investor.

- Money market investments are very safe; they almost never go in the red, but they also don't pay high returns.
- Stocks are on the opposite end of the spectrum, going back and forth between red and black from year to year frequently, but over longer periods of time they usually pay higher premiums.
- Bonds are somewhere in the middle. They are safer than a stock, but riskier than a money market and their average returns reflect that.

Calculating variance is a 3-step process once the expected return has been calculated.

STEP BY STEP

1. Calculate deviations from mean or expected value
2. Square the deviations
3. Multiply the squared deviation by its original probability

You may not need to calculate variance yourself, but you should still understand how we got it.

Let's go back to the scenario from above.

IN CONTEXT

Recall that you are considering investing money into stock of a ski/snowboard mountain resort in Colorado. We started with three winter scenarios and a probability (P) and return (R) associated with each. We also found an expected return of 9.25%. This table shows how to calculate the variance of an investment outcome.

Scenario	P	R	(R - E(R))	$(R - E(R))^2$	$(P)(R - E(R))^2$
SG	0.25	25%	$(25\% - 9.25\%) = 15.75\%$	$(15.75\%)^2 = 248.0625$	$(0.25)(248.0625) = 62.02$
TC	0.60	10%	$(10\% - 9.25\%) = 0.75\%$	$(0.75\%)^2 = 0.5625$	$(0.60)(0.5625) = 0.34$
WB	0.15	-20%	$(-20\% - 9.25\%) = -29.25\%$	$(-29.25\%)^2 = 855.5625$	$(0.15)(855.5625) = 128.33$
	Variance $= 62.02 + 0.34 + 128.33 = 190.69$			Standard Deviation $= \sqrt{190.69} = 13.81\%$	

First, we found deviations by subtracting the expected return from each stock return. Then we squared this deviation. Finally, we multiplied each squared deviation by the probability.

Of the three numbers, we ended up with 62, 0.34, and 128. Two are very big and one is very small. The small number comes from the TC scenario where the stock returns 10%, which is very close to our expectation of 9.25%. The bigger numbers come from winters that are extreme — when the stock performs way above 9.25% (HG) or way below it (WB).

The **standard deviation** can be read as a percentage. It means that even though we can expect an average of 9.25% return on our stock over the course of 50 years, if we take any given year out and look at its performance, it is likely be somewhere within 13.81% above or below that figure.

TERM TO KNOW

Variance

Used to measure the degree of risk in an investment. It is calculated by finding the average of the squared deviations from the mean rate of return.

Standard Deviation

Obtained by taking the square root of the variance. It has a more straightforward meaning than variance. It tells you that in a given year, you can expect an investment's return to be one standard deviation above or below the average rate of return.

3. Variance in Relation to Expected Return

In the discussion of expected return, we concluded that, based on your research, you can expect the ski/snowboard resort in Colorado to have an expected return of 9.25% based on three distinct weather outcomes. However, if you invest your $20,000 in that company and expect to have $21,850 after a year, you must remember that this isn't a dice game that you can play over and over again. There will only be one result in this case and at the end of it, you have to make a down payment on a house. Is this a good investment idea?

What if your bid for a house won't be accepted unless you can put at least $20,000 down? There is an 85% chance that the winter is either super gnarly (SG) or totally chillax (TC), and in either of those cases you will still have over $20,000 to make a down payment. There is also a 15% chance that the year ends up being wicked bogus (WB), and if that is the case, you will lose 20% or $4,000 of your initial investment. Now you have $16,000.

Let's compare that investment to a CD at a bank that pays 3.25% no matter how much snow falls this winter. You can have an investment that is federally insured to pay you $20,650 one year from today and you can be assured to have enough to make a down payment on your house.

If you can let that $20,000 investment sit for 10 years before you need it, there is a much better chance that you will end up in the black (experiencing a profit) than in the red (experiencing a loss). A 30-year-old with a 401K can be much more aggressive in his portfolio than a 65-year-old who will be retiring in one year. It would be just as foolish for the 65-year-old to be investing in aggressive stocks as it would for the 30-year-old to buy conservative CD's in his retirement account.

BIG IDEA

Every portfolio should be modeled with time-frame and risk tolerance considerations.

SUMMARY

In this lesson, you learned how to determine **expected return**, by considering possible investment outcomes and the probability of each occurring. You also learned about **variance**, which is a statistical measure of how far investment outcomes fall from the expected return. Understanding **variance in relation to expected return** provides insight into the risk of an investment and allows you to make informed investment choices.

Best of luck in your learning!

TERMS TO KNOW

Expected Return
Computed by computing the product of the probability of a given event and the return in that case and adding together the products in each discrete scenario.

Standard Deviation
Obtained by taking the square root of the variance. It has a more straightforward meaning than variance.

It tells you that in a given year, you can expect an investment's return to be one standard deviation above or below the average rate of return.

Variance

Used to measure the degree of risk in an investment. It is calculated by finding the average of the squared deviations from the mean rate of return.