Exponential Growth

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WHAT'S COVERED

In this lesson, you will learn how to calculate a growth amount by solving for *y* in the exponential equation. Specifically, this lesson will cover:

1. Exponential Growth Formula

The general exponential equation is $y = a \cdot b^x$, where a base number, *b*, is raised to a variable exponent power, *x*. There is also a scalar multiplier, a, in front of the exponential expression.

Exponential growth can be modeled with a similar equation:

FORMULA TO KNOW

Exponential Growth

$y = a(1+b)^x$

In this formula,

- a is the initial value.
- *b* is the rate of change.
- *x* is the number of time intervals.

With exponential growth, *b* represents the rate of change (expressed as a decimal), and is added to one to indicate growth. We can call $(1+b)$ the growth factor, since it's multiplied by a (the initial value) *x* number of times.

2. Calculating a Growing Population

In 2000, the population of Berlin was about 3.39 million. The city's population has been growing at an average rate of 0.27% each year. Assuming this rate of growth remains the same, what is Berlin's expected population in the year 2050?

From the scenario, we can draw the following values for certain variables in the exponential growth formula:

- $a = 3.39$ (expressed in millions, the population in the year 2000)
- $b = 0.0027$ (0.27% expressed as a decimal)
- $x = 50$ (50 years between 2000 and 2050)

Substitute these values into the exponential growth formula, $y = a(1+b)^x$, to find the population after 50 years, *y*:

$y = a(1+b)^x$	Plug in $a = 3.39$, $b = 0.0027$, $x = 50$
$y = 3.39(1+0.0027)^{50}$	Simplify inside the parentheses
$y = 3.39(1.0027)^{50}$	Apply exponent
$y = 3.39(1.144328)$	Multiply
$y = 3.88$	Our solution

The expected population for Berlin in the year 2050 is about 3.88 million people.

HINT

It is helpful to include as many digits as possible during calculations, and then round at the end. This is especially helpful when your solution is a large number, such as a population in the hundreds of millions.

3. Solving for Growth Time

We can also use the exponential growth formula to calculate for a period of time. A common application of exponential growth is the spread of viruses.

During the 14th century, the "Black Death" devastated the population of Europe. At its peak in 1350, the plague had infected 1.5 million in just two years. If the plague had continued to spread at a rate of 18.27%, how long would it have taken to infect 4 million people?

Looking at the exponential growth formula, we no longer have a known value for *x*, as *x* represents time. However, we can make the following substitutions:

- $y = 4$ (expressed in millions, ending value)
- $a = 1.5$ (expressed in millions, initial value)
- $r = 0.1827$ (18.27% expressed as a decimal)

Solving for x in this case is going to require that we use logarithms, because it is the inverse operation of exponents:

$y = a(1+b)^x$	Plug in $y = 4,\ a = 1.5,\ b = 0.1827$
$4 = 1.5(1+0.1827)^x$	Simplify inside the parentheses
$4 = 1.5(1.1827)^x$	Divide by 1.5
$2.67 = 1.1827^x$	Apply log to both sides
$\ln(2.6667) = \ln(1.1827^x)$	Apply Power Property of Logs
$\ln(2.6667) = x \cdot \ln(1.1827)$	Divide both sides by $\ln(1.1827)$
$\frac{\ln(2.6667)}{\ln(1.1827)} = x$	Evaluate fraction
$5.8 = x$	Our solution

It would take about 5.8 years for the plague to infect 4 million people.

HINT

We could have applied either the common log (log) or natural log (ln) function to undo the exponent.

BIG IDEA

There are a couple of things to note when solving these types of problems:

- Before doing anything about the variable exponent, divide the equation by the *a*-value. This is because the exponent is attached to the expression inside the parentheses $(1+b)$, not a.
- You can apply the log of any base to undo the exponent, so long as the same base is used on both sides of the equation. More often than not, you will use either natural log (base e) or common log (base 10); either of which is fine.
- In applying the log to both sides, we use the power property of logarithms to bring the exponent outside of the log function, and place it as a scalar multiplier in front of the log. This allows us to isolate the variable through division.

SUMMARY

Exponential growth equations can be used to represent real world situations, such as population growth or the spread of a virus. In the **exponential growth formula**, a is the initial value and b is the rate of change expressed as a decimal. It is added it 1 to represent growth. When **calculating a growth population**, you are solving for y. When **solving for growth time**, you are solving for the variable exponent. One method is to take the log of both sides of the equation and apply properties of logs.

FORMULAS TO KNOW

Exponential Growth

$y = a(1+b)^x$