Finding the Intersection Point of Two Lines

by Sophia

WHAT'S COVERED

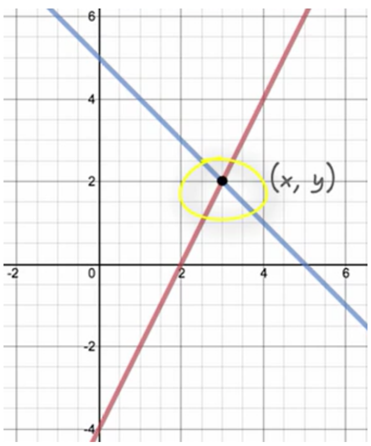
This tutorial covers finding the intersection point of two lines, through the exploration of:

1. System of Equations

Systems of equations have applications in economics, physics, and engineering. A system of linear equations consists of two or more equations with the same variables considered at the same time. This means that the equations in a system are graphed together on a single coordinate plane. The solution to this system of equations is the solution that satisfies all individual equations in the system.

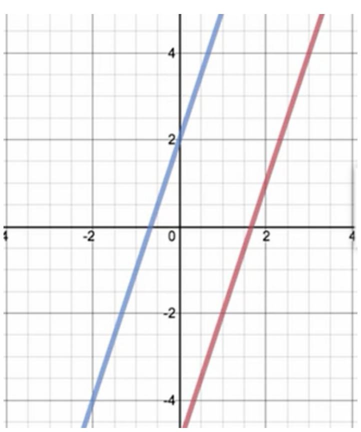
EXAMPLE Below you see a system of equations and its graphs. The point of intersection of the lines represents the solution to the two equations of the lines. The intersection point represents an x value and y value that can be substituted into each equation in the system to yield a true statement.

$y = 2x - 4$
$y = -x + 5$



EXAMPLE In this next example, you see a graph representation of the system of equations y equals 3x minus 5 and y equals 3x plus 2. These two lines are parallel, meaning they do not have an intersection point, which in turn means there is no solution to the system of equations.

$y = 3x - 5$
$y = 3x + 2$



TERM TO KNOW

System of Linear Equations
Two or more equations with the same variables, considered at the same time

2. Graphing a System of Equations

It is possible to solve a system of equations by graphing the equations in the system, then finding the intersection point of the lines.

IN CONTEXT

Suppose a company is analyzing supply and demand data for their product. You can utilize a system of equations that represent the price of the product in relation to the quantity supplied and the quantity demanded, in order to represent the situation. In the following equations, the first equation represents the demand for the product, while the second equation represents the supply of the product.

$y=-\frac{1}{2}x+60$ *Demand*

$y=\frac{3}{2}x$ *Supply*

To solve, you can graph both equations and find the intersection point. In the first equation, the y-intercept is 60. Therefore, you have a point at (0, 60). Next, use your slope, which is -1 over 2, to find your second point. Since the graph uses intervals of 5, you move down 5 and over 10 from the y-intercept to place your second point. Lastly, connect both points with a line.

In the second equation, the y-intercept is 0 because there is no written b value. Therefore, you have a point at the origin (0, 0). Use your slope, which is 3 over 2, to find your second point. From your y-intercept, you move up 3 and over 2 to place your second point. Lastly, connect both points with a line.

You can see that your lines intersect at the point (30, 45), which means that the solution to the system is x equals 30 and y equals 45. This is where the supply and demand are equal, and the company will maximize their profits by setting the price at $30 and selling a quantity of 45 units.

You can verify your solution by substituting it into both equations to see if they yield true statements, as shown below. Simplifying on both sides provides 45 equals 45; therefore, your solution of x equals 30 and y equals 45 is correct.

$$45 = -\frac{1}{2}(30) + 60 \qquad 45 = \frac{3}{2}(30)$$
$$45 = -15 + 60 \qquad 45 = 45$$
$$45 = 45$$

TRY IT

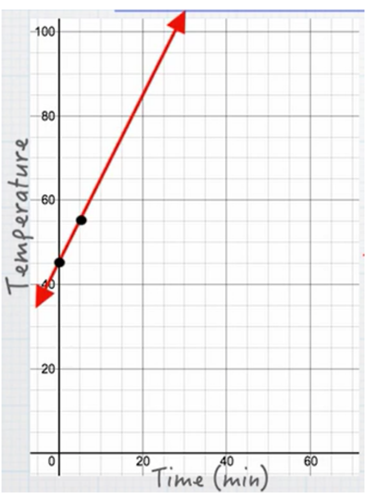
Now that you know how to graph a system of equations and find the intersection point, use your knowledge to solve the following problem. Suppose a company is testing a new product. While the product is running, they measure the temperature every minute. They need to make sure the temperature stays below 85 degrees. You can represent this situation with a system of equations. The first equation represents the temperature of the machine, and the second equation represents the maximum temperature that can be reached.

$y = 2x + 45$ *Temperature of product*
$y = 85$ *Max temperature*

To solve, you need to graph both equations and find the intersection point.

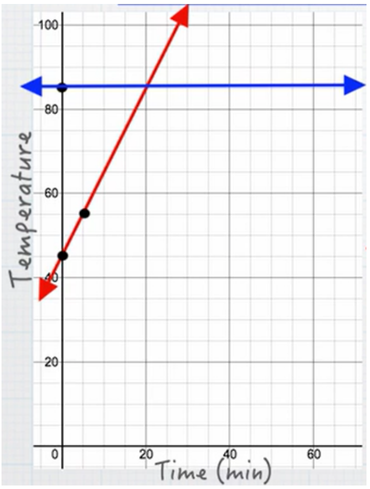
Graph the first equation for the temperature of the machine. +

In the first equation, the y-intercept is 45, so you know that your first point is (0, 45). Next, you can see that your slope is 2, which is the same as 10 over 5, so on your graph, you can move up to 10 and over 5 to place your second point. Finally, connect both points with the line.



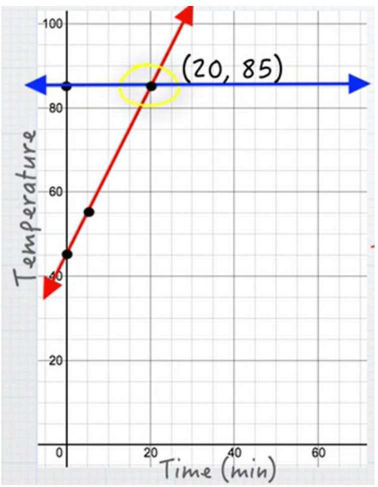
Graph the second equation for max temperature. +

The second equation is a horizontal line that goes through 85 on the y-axis, so you can plot a point at (0, 85) and draw a horizontal line through the point.



Use the intersection point to find the solution. +

You can see that these two lines intersect at the point (20, 85), which means that the solution to your system is x equals 20 and y equals 85. This also means that it will take 20 minutes for the temperature to reach 85 degrees.



Remember, you can verify this by substituting the solution into both equations to see if they yield true statements. Doing this in both equations and simplifying provides a true statement of 85 equals 85. Therefore, your solution x equals 20 and y equals 85 is correct.

$$85 = 2(20) + 45$$
$$85 = 40 + 45$$
$$85 = 85$$

SUMMARY

Today you learned about systems of equations, which consist of two or more equations with the same variables considered at the same time. You also learned that the solution to a system of equations is a solution that satisfies all individual equations in the system. Finally, you learned that by graphing a system of equations and finding the point of intersection, you can solve the system of equations.

Source: This work is adapted from Sophia author Colleen Atakpu.

TERMS TO KNOW

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