Finding the Inverse of a Function

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WHAT'S COVERED

In this lesson, you will learn how to determine the inverse of a given function. Specifically, this lesson will cover:

1. Inverse Functions

An inverse function undoes the operations performed on variables of a function.

EXAMPLE If a number is multiplied by 2, and added by 3, we can write this as the function $f(x) = 2x + 3$. The inverse to this function first subtracts 3 from the input value, and then divides by 2, so as to completely undo all operations of the original function. We write this as the inverse function $f^{-1}(x) = \frac{x-3}{2}$.

BIG IDEA

If a function becomes the input of an inverse function, then the output is the argument of the original function. Mathematically, we write this as: $f^{-1}(f(x)) = x$

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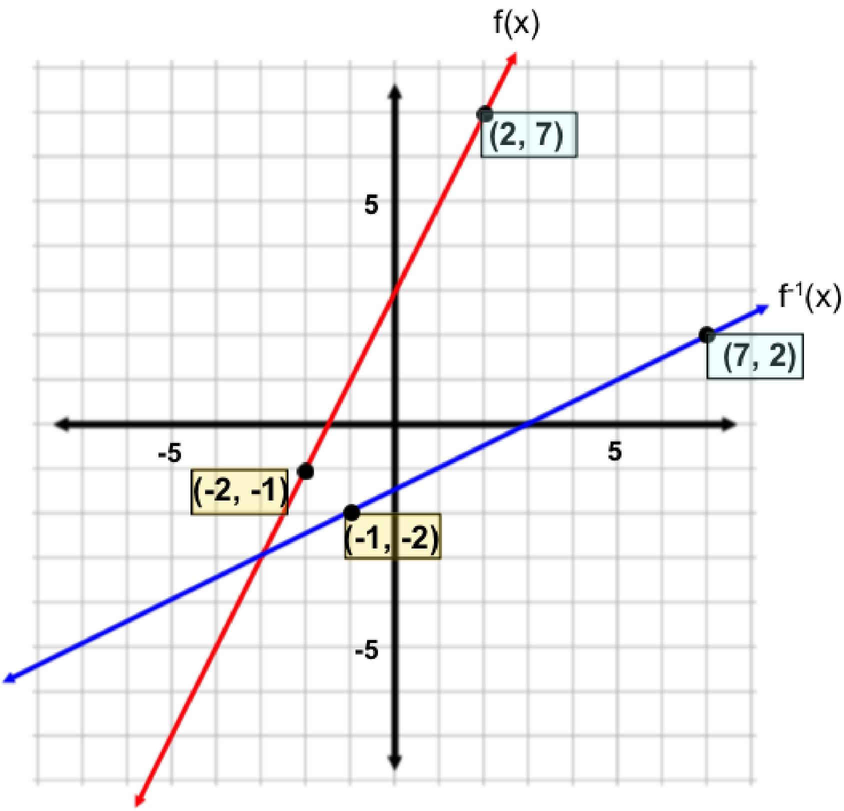
2. Evaluating an Inverse Graphically

On a graph, the x- and y-coordinates between a function and its inverse are inverted or swapped. This means that for any coordinate, (x, y), of a function, we can find a corresponding coordinate on the graph of its inverse using the coordinates (y, x). This means we locate the x-value on the y-axis, and locate the y-value on the x-axis.

EXAMPLE Check out the graph of a function and its inverse.



Points on $f(x)$	Points on $f^{-1}(x)$
(2, 7)	(7, 2)
(-2, -1)	(-1, -2)

3. Finding the Inverse Algebraically

If we want to find the inverse of a function algebraically, there are two common procedures most people use:

1. Rewrite the equation, except swap x and y. Then, rewrite the equation so that y is isolated on one side of the equation.
2. Do the same process, but in reverse order. First, you can rewrite the equation so that x is isolated on one side of the equation. Then, simply swap x and y. The resulting equation will be the defined inverse function.

↱ EXAMPLE Find the inverse of $f(x)=\sqrt{2x-4}$ using the first method where we swap x and y.

$f(x)=\sqrt{2x-4}$	Rewrite the function as $y=$
$y=\sqrt{2x-4}$	Swap x and y
$x=\sqrt{2y-4}$	Square both sides
$x^2=2y-4$	Add 4 to both sides

$x^2+4=2y$	Divide both sides by 2
$\frac{1}{2}x^2+2=y$	Our solution for y
$f^{-1}(x)=\frac{1}{2}x^2+2$	Our solution in inverse notation

↪ EXAMPLE Find the inverse of $f(x)=\frac{x+7}{3}$.

$f(x)=\frac{x+7}{3}$	Rewrite the function as $y=$
$y=\frac{x+7}{3}$	Swap x and y
$x=\frac{y+7}{3}$	Multiply both sides by 3
$3x=y+7$	Subtract 7 from both sides
$3x-7=y$	Our solution for y
$f^{-1}(x)=3x-7$	Our solution in inverse notation

HINT

Technically, we need to restrict the domain of the inverse function to non-negative values of x. This is because the range of the original function was restricted to non-negative y-values. The domain and range of a function also swap when defining the domain and range of its inverse.

SUMMARY

The **inverse of a function** undoes the operations of the function. We can **evaluate an inverse graphically** by comparing the coordinate points. All points on the curve of $f(x)$ can be described as (x, y). All points on the curve inverse of $f(x)$ can be described as (y, x), where x and y are the coordinates of the original function. The **inverse can be found algebraically** by swapping x and y, and then solving the equation for y.