Finding the Slope of a Line on a Graph

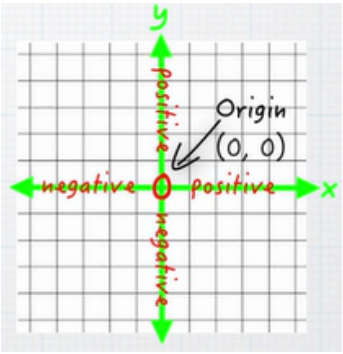
by Sophia

WHAT'S COVERED

This tutorial covers slope on a line graph, through the definition and discussion of:

1. Coordinate Plane: A Review

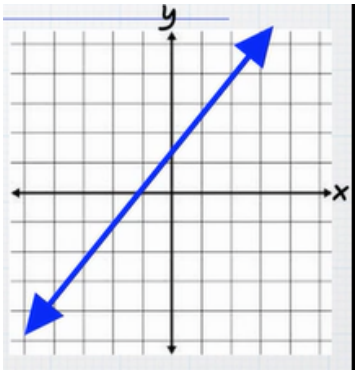
A coordinate plane has a horizontal and a vertical axis. The horizontal axis is the x-axis. It has a positive side and a negative side and is centered at zero. The vertical axis is the y-axis, and it also has a positive side and a negative side, and is centered at zero. The intersection of the two axes is known as the origin. Points on the plane are written as an ordered pair (x, y), where x is the x-coordinate, y is the y-coordinate, and the origin is at (0, 0).



2. Slope

The steepness of a line is called its **slope**. The slope of a line can be calculated using the x and y coordinates of any two points on the line.

Here is an example of a straight line that extends infinitely in both directions.



To calculate the slope, divide the change in y-coordinates by the change in x-coordinates from any two points on the line.

FORMULA TO KNOW

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- In this formula, m is the variable that is used most commonly to represent the slope of a line.
- The 2s and 1s in the formula relate to the x- and y-coordinates of the two points.

$(x_1, y_1) = \textit{the first point}$
$(x_2, y_2) = \textit{the second point}$
$y_2 - y_1 = \textit{the difference between the two } y - \textit{coordinates}$
$x_2 - x_1 = \textit{the difference between the two } x - \textit{coordinates}$

HINT

Another way to think about slope is the change in y over the change in x or rise over run, because rise describes the change in y, a vertical change, and run describes the change in x, a horizontal change.

TERM TO KNOW

Slope

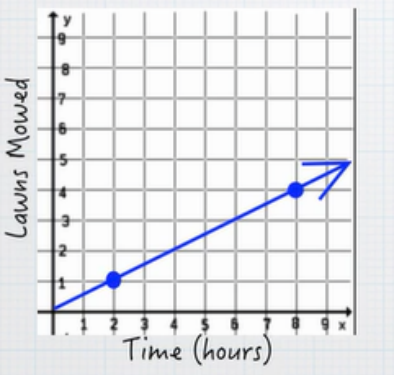
The steepness of a line; found by dividing the change in y-coordinates by the change in x-coordinates from any two points on a line

3. Calculating Slope

Calculating slope is useful in many everyday situations, including price and cost, transportation fares, and inclines, such as roof tops, ski slopes, and parking ramps.

3a. Positive or Negative Slopes

EXAMPLE Suppose you have a landscaping business. The line below represents the relationship between time in hours and the number of lawns mowed.



You can pick any two points on the line to calculate the slope, such as (2,1) and (8,4). Make sure you label your points as shown below.

$(x_1, y_1) = (2,1)$
$(x_2, y_2) = (8,4)$

Substitute these values into the formula and simplify:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{8-2} = \frac{3}{6} = \frac{1}{2}$$

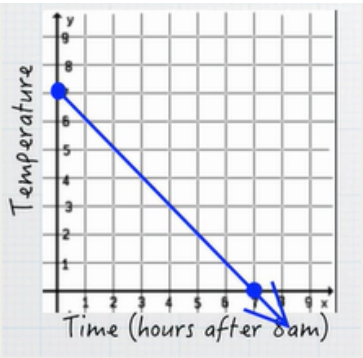
Therefore, 1/2 is the slope of the line between any two points on this graph. It also means that one lawn takes two hours to mow.

$x_1\ y_1 \quad x_2\ y_2$
$(2,\ 1) \quad (8,\ 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{8-2} = \frac{3}{6} = \frac{1}{2}$$

IN CONTEXT

Suppose the temperature is dropping throughout the day. The line below represents the relationship between time in hours after 8:00 a.m. and the temperature. By looking at the line, you can see that the slope will be negative because the line goes down as you read the graph from left to right. Can you calculate the slope?



To calculate the slope, use the two points (0, 7) and (7, 0) and label them in accordance with the slope formula. Substitute these values into the formula and simplify.

$x_1\, y_1 \quad x_2\, y_2$

$(0, 7) \quad (7, 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = \frac{-7}{7} = -1$$

Therefore, -1 is the slope of the line between any two points on this graph, which also means that the temperature is decreasing by 1 degree each hour after 8 a.m.

3b. Zero Slope

The next example illustrates a case in which the lines either have *no* steepness.

EXAMPLE The graph below shows the height of a teenager in feet, in relation to his or her age in years after 18. By looking at the line, you can see that the line has a 0 slope, meaning no steepness, because it is a horizontal line. This means there is zero change in the values of the y-coordinates.

To calculate the slope, use the points (2, 6) and (6, 6), and label them accordingly. Substitute these values into the formula and simplify.

$$(x_1, y_1) = (2, 6)$$
$$(x_2, y_2) = (6, 6)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-6}{6-2} = \frac{0}{4} = 0$$

Note that this simplifies to 0 over 4 or 0 divided by 4, which is 0. Therefore, 0 is the slope of the line between any two points on the graph because there is no change in the y-coordinates between any two points. This also means that the person's height is not changing over time after age 18.

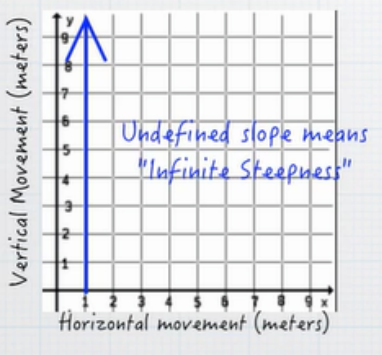
BIG IDEA

All horizontal lines have a slope of zero because there is no change in the y-coordinates between any two points on the line.

3c. Undefined Slope

The next example illustrates a case in which the line has *infinite* steepness.

EXAMPLE This graph represents a very steep part of a cliff, illustrating the vertical movement as it relates to the horizontal movement of a climber. By looking at the line, you can see that the line has an undefined slope, meaning infinite steepness because it is a vertical line. This means that there is zero change in the values of the x-coordinates.



To calculate the slope, use the points (1, 1) and (1, 2), and label them accordingly. Substitute these values into the formula and simplify.

$$(x_1, y_1) = (1, 1)$$
$$(x_2, y_2) = (1, 2)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{1-1} = \frac{1}{0}; \text{ undefined}$$

This simplifies to 1 over 0, which is undefined because you cannot have 0 in the denominator of a fraction, because we cannot divide by zero. Therefore, the slope is undefined between any two points on the graph, because there is zero or no change in the x-coordinates between any two points.

BIG IDEA

All vertical lines have a slope that is undefined because there is no change in the x-coordinates between any two points.

SUMMARY

Today you reviewed the concept of a **coordinate plane**, then learned about the steepness of a line, which is called its **slope**. You also learned how to find the slope of a line on a graph, and practiced **calculating the slopes** of lines with a positive, negative, zero, and undefined slope.

Source: This work is adapted from Sophia author Colleen Atakpu.

TERMS TO KNOW

Slope

The steepness of a line; found by dividing the change in y-coordinates by the change in x-coordinates from any two points on a line.

FORMULAS TO KNOW

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$