Fractional Exponents and Radicals

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WHAT'S COVERED

In this lesson, you will learn how to write fractional exponents as radicals, or vice versa. Specifically, this lesson will cover:

1. Inverse Operations: Radicals and Exponents

Just as multiplication and division are inverse operations of one another, radicals and exponents are also inverse operations. For example, suppose we have the number 3 and we raise it to the second power. Now if we were to take the square root of 3^2, notice that we will end up with 3. This is the number with which we started.

$$\sqrt{3^2} = 3$$

Below are a few more examples:

$$\sqrt[3]{4^3} = 4$$

$$\sqrt[4]{2^4} = 2$$

$$\sqrt[9]{8^9} = 8$$

$$\sqrt{5^2} = 5$$

2. Converting Radicals to Exponents

Any radical can be rewritten as an exponent by using Rule #1 of **properties of fractional exponents**.

FORMULA TO KNOW

Properties of Fractional Exponents

Rule #1: $\sqrt[m]{a} = a^{\frac{1}{m}}$

Let's look at a few examples:

$$\sqrt{b} = b^{\frac{1}{2}}$$

$$\sqrt[3]{c} = c^{\frac{1}{3}}$$

$$\sqrt[7]{f} = f^{\frac{1}{7}}$$

HINT

A common radical conversion to exponent is square root. Remember that taking the square root of a term is the same as raising that term to the 1/2 power.

3. Fractional Exponents and Radicals

Whenever we are working with the exponents, taking the appropriate radical will always cancel out the exponent operation. That is to say, if a number is raised to the power of a, we can cancel out the exponent by taking the resulting value and taking the a-th root of the result. This is the same as raising the resulting number to a fractional exponent.

EXAMPLE If we have 7^4, in order to cancel out the exponent 4 we would have to take the 4th root of 7^4 which is the same as raising 7^4 to the ¼ power. This is shown below:

$$\sqrt[4]{7} = (7^4)^{\frac{1}{4}} = 7^{\frac{4}{4}} = 7$$

In general, whenever we have an expression raised to the a-th power, we can take the a-th root of that number, which is the same as raising that number to the $\frac{1}{a}$ power. By doing this, we effectively cancel out the radical with the exponent.

$$\sqrt[a]{x^a} = (x^a)^{\frac{1}{a}} = x^{\frac{a}{a}} = x^1 = x$$

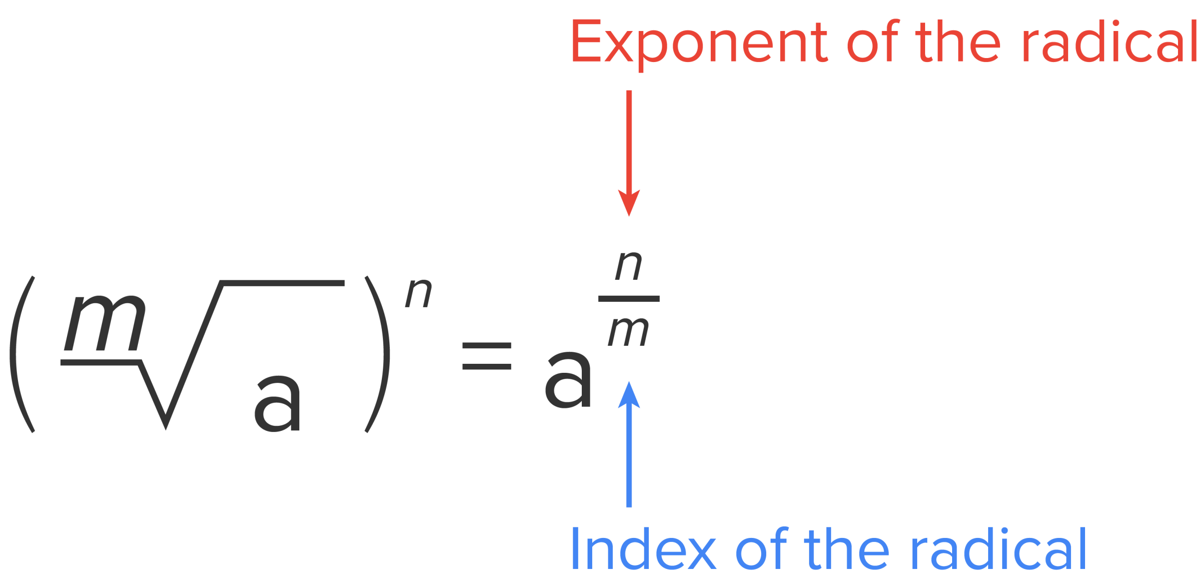
Let’s look at this a little more in-depth. When we simplify radicals with exponents, we divide the exponent by the index. Another way to write division is with a fraction bar. This is how we will define rational exponents.

FORMULA TO KNOW

Properties of Fractional Exponents

Rule #2: $(\sqrt[m]{a})^n = a^{\frac{n}{m}}$

The denominator of a rational exponent becomes the index on our radical. Likewise, the index on the radical becomes the denominator of the exponent. We can use this property to change any radical expression into an exponential expression.



3a. Converting to Fractional Exponents

Let’s look at some examples of converting radicals to fractional exponents.

Radical	Fractional Exponent	Explanation
$\sqrt{y^3}$	$y^{\frac{3}{2}}$	To convert $\sqrt{y^3}$ to a fractional exponent, we identify the index and the exponent of the radical. If no index is given, it is assumed to be 2 or the square root. The exponent of this radical is 3. The numerator is 3 and the denominator of the fraction is 2. So, the fractional exponent is $y^{\frac{3}{2}}$.
$\sqrt[4]{a}$	$a^{\frac{1}{4}}$	To convert $\sqrt[4]{a} = a^{\frac{1}{4}}$ to a fractional exponent, we identify the index and the exponent of the radical. The index of the radical is 4. If no exponent is given, it is assumed to be 1. The numerator is 1 and the denominator of the fraction is 4. So, the fractional exponent is $a^{\frac{1}{4}}$.
$(\sqrt[5]{x})^3$	$x^{\frac{3}{5}}$	The exponent of this radical and the numerator of the radical is 3. The index of the radical and the denominator of the fraction is 5. So, the fractional exponent is $x^{\frac{3}{5}}$.
$(\sqrt[6]{3x})^5$	$(3x)^{\frac{5}{6}}$	The exponent of this radical and the numerator of the radical is 5. The index of the radical and the denominator of the fraction is 6. Note the expression under the radical $3x$ remains the base of the fractional exponent. So, the fractional exponent is $(3x)^{\frac{5}{6}}$.

TRY IT

Consider the expression $(\sqrt{(4y)})^3$.

Convert this expression to a fractional exponent. +

The exponent of this radical and the numerator of the fractional exponent is 3. The index of the radical and the denominator of the fraction is 2. Note the expression under the radical $(4y)$ remains the base of the fractional exponent. So, the fractional exponent is $(\sqrt{(4y)})^3 = (4y)^{\frac{3}{2}}$.

3b. Converting to Radicals

Let's look at some examples of converting fractional exponents to radicals.

Fractional Exponent	Radical	Explanation
$7^{\frac{3}{4}}$	$(\sqrt[4]{7})^3$	To convert $7^{\frac{3}{4}}$ to a radical, the numerator of the fractional exponent, 3, is the exponent. The denominator of the exponent is the index of the radical, 4. So, the radical is $(\sqrt[4]{7})^3$.
$(2mn)^{\frac{2}{7}}$	$(\sqrt[7]{2mn})^2$	To convert $(2mn)^{\frac{2}{7}}$ to a radical, the numerator of the fractional exponent, 2, is the exponent of the radical. The denominator of the fraction, 7, is the index of the radical. Notice the base of the exponent remains under the radical. So, the radical is $(\sqrt[7]{2mn})^2$.
$a^{\frac{5}{3}}$	$(\sqrt[3]{a})^5$	To convert $a^{\frac{5}{3}}$ to a radical, the numerator of the fractional exponent, 5, is the exponent of the radical. The denominator of the exponent 3 is the index of the radical. So, the radical is $(\sqrt[3]{a})^5$.

TRY IT

Consider the expression $(3n)^{\frac{3}{5}}$.

Convert this expression to a radical. +

To convert $(3n)^{\frac{3}{5}}$ to a radical, the numerator of the fractional exponent, 3, is the exponent of the radical. The denominator of the fraction, 5, is the index of the radical. Notice the base of the exponent, $(3n)$, remains under the radical. So, $(3n)^{\frac{3}{5}} = (\sqrt[5]{3n})^3$.

SUMMARY

Exponents and radicals are **inverse operations** of each other, meaning they cancel each other out. **Fractional exponents and radicals** can be written as one another by using the property of fractional exponents. The largest advantage of being able to change a radical expression into an exponential expression is that we are now allowed to use all of our exponent properties to simplify.

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FORMULAS TO KNOW

Properties of Fractional Exponents

Rule #1: $\sqrt[m]{a} = a^{1/m}$

Rule #2: $(\sqrt[m]{a})^n = a^{n/m}$