Function of a Function

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WHAT'S COVERED

In this lesson, you will learn how to evaluate a composite function. Specifically, this lesson will cover:

1. Composite Functions

You may have noticed that when we evaluate a function, we typically evaluate it for a number or variable, but we can go one step further and evaluate a function of another function. This process is very similar to when you would substitute a variable with an expression.

EXAMPLE Substitute $y = 3x + 1$ into $2x - y = 4$ and solve for *x*.

Since $y = 3x + 1$, we can substitute $3x + 1$ in for y into $2x - y = 4$.

$2x - y = 4$	Substitute $3x + 1$ in for *y*
$2x - (3x + 1) = 4$	Distribute subtraction
$2x - 3x - 1 = 4$	Combine like terms
$-x - 1 = 4$	Add 1 to both sides
$-x = 5$	Multiply both sides by -1
$x = -5$	Our Solution

When working with **composite functions**, we follow a similar process. If we are given two functions $f(x)$ and $g(x)$ and we want to find $f(g(x))$ we simply replace all instances of *x* in the function $f(x)$ with whatever $g(x)$ is set equal to. Then we simplify.

BIG IDEA

When we have a composite function, the notation $f(g(x))$ can be written as $(f \circ g)(x)$. This expression is read, "*f* of *g* of *x*" and means that we are replacing the *x* values of inputs of $f(x)$ with another function, in this case, $g(x)$.

TERM TO KNOW

Composite Function

The combination of functions, such that the output of one function is the input of another.

2. Evaluating a Composite Function

Let's take a look at how to actually evaluate a composite function.

↱ EXAMPLE Suppose $f(x) = x^2 - 1$ and $g(x) = x - 1$. Find $f(g(x))$.

The composite function $f(g(x))$ means we will replace all instances of x in $f(x)$ with the function $g(x)$, which is $g(x) = x - 1$. To do this we can take the following steps:

$f(x) = x^2 - 1$	Replace x with $g(x)$
$f(g(x)) = (g(x))^2 - 1$	Replace $g(x)$ with $x - 1$
$f(g(x)) = (x - 1)^2 - 1$	Expand $(x - 1)^2$
$f(g(x)) = (x - 1)(x - 1) - 1$	FOIL
$f(g(x)) = (x^2 - 2x + 1) - 1$	Evaluate
$f(g(x)) = x^2 - 2x$	Our solution

Sometimes we may be asked to solve a particular composite function when x is equal to a given value, for example $x = 2$. To make this evaluation, we follow the same steps we did above then at the end we simply substitute the value we are given for x. Let's see how we do this:

↱ EXAMPLE Using the result from above, evaluate $f(g(2))$.

Our result for $f(g(x))$ was $f(g(x)) = x^2 - 2x$. Now substitute 2 in for x.

$f(g(x)) = x^2 - 2x$	Replace x with 2
$f(g(2)) = (2)^2 - 2(2)$	Evaluate
$f(g(2)) = 4 - 4$	Simplify
$f(g(2)) = 0$	Our solution

Alternatively, we could have evaluated $g(2)$ first and then plug that value in for x in the function $f(x)$. Either method would give us the same result.

$g(x) = x - 1$	Evaluate $g(2)$
$g(2) = 2 - 1$	Simplify
$g(2) = 1$	Evaluate $f(g(2))$ by plugging in 1 for $g(2)$
$f(g(2)) = f(1)$	Evaluate $f(1)$
$f(1) = 1^2 - 1$	Simplify
$f(1) = 0$	Our Solution

HINT

Sometimes you may be asked to find $g(f(x))$. In this case, we do the same procedures only we replace each variable in $g(x)$ with the expression for $f(x)$.

3. Evaluating *f*(*f*(*x*))

Sometimes you may come across a situation where you will need to evaluate a function for itself, namely $f(f(x))$. This simply means that you are plugging in the expression of the function for each *x* in the function.

EXAMPLE Find $f(f(x))$ for $f(x) = 2x - 1$.

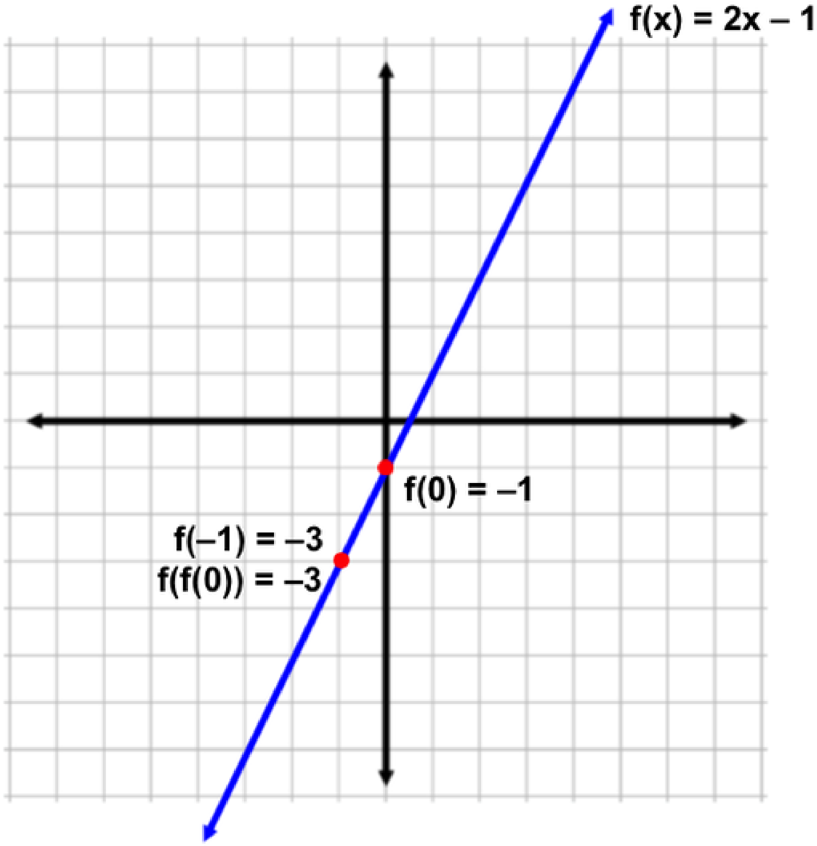
$f(x) = 2x - 1$	Replace *x* with $f(x)$
$f(f(x)) = 2(f(x)) - 1$	Replace $f(x)$ with $2x - 1$ on the right side
$f(f(x)) = 2(2x - 1) - 1$	Distribute 2 into $2x - 1$
$f(f(x)) = 4x - 2 - 1$	Simplify
$f(f(x)) = 4x - 3$	Our solution

Like with the previous problem, you may be asked to find the composite function of itself when *x* is a given value, for example $f(f(0))$. Just like before we simply need to replace each instance of *x* in the function $f(f(x))$ with the given value.

EXAMPLE Using the result from above, evaluate $f(f(0))$.

$f(f(x)) = 4x - 3$	Replace *x* with 0
$f(f(0)) = 4(0) - 3$	Evaluate
$f(f(0)) = 0 - 3$	Simplify
$f(f(0)) = -3$	Our solution

We can show this in the graph below. First note that if $f(x) = 2x - 1$, then $f(0) = -1$. Plugging in -1 in for *x* in $f(x)$ we get $f(-1) = -3$, which is the same answer we found doing this algebraically.



SUMMARY

A **composite function** is a function of a function. We used the notation $(f \circ g)(x)$, which is read as "*f* of *g* of *x*". When **evaluating a composite function**, the output of the innermost function becomes the input of the outermost function. It is not always true that $(f \circ g)(x)$ is equal to $(g \circ f)(x)$. **Evaluating *f*(*f*(*x*))** simply means that we are plugging in the expression of the function for each *x* in the function.

TERMS TO KNOW

Composite Function
The combination of functions, such that the output of one function is the input of another.