Geometric Distribution

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WHAT'S COVERED

In this tutorial, you're going to learn about the geometric distribution. Our discussion breaks down as follows:

1. Geometric Distribution

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The **geometric distribution** is somewhat similar to the binomial distribution. It's a probability distribution with a particular setting; it has a geometric setting.

A scenario or an experiment can be considered geometric if it fits the following four criteria:

1. Every trial of the chance experiment only has two outcomes: success or failure.
2. Every trial has to be independent of each other, which means that the result of one trial doesn't affect the probabilities for any other.
3. You need a fixed probability of success on every trial (p).
4. The variable of interest--what you're looking to measure--is the number of trials needed in order to achieve your first success.

HINT

Success is fairly arbitrary, just like with the binomial. You can rig an experiment to make it two outcomes, even if there's not. For instance, on a die, you could say rolling a one is a success, and everything else is a failure. Even though there are six possible outcomes with rolling a die, you can rig it so that one event counts as a success and some event counts as a failure. The only requirement is that these two be complementary events.

You can liken this to the lottery because you're only going to play until you win. You essentially play and lose all the way up until you win, and then you stop.

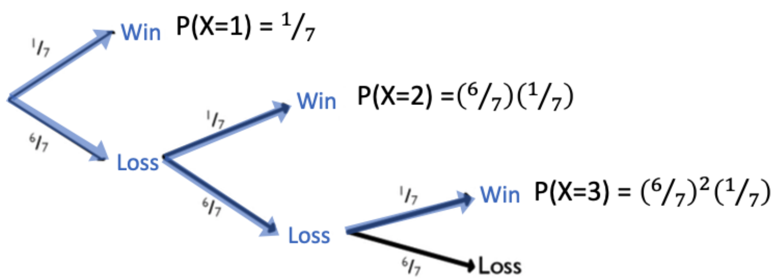
EXAMPLE Suppose a soda company is running a promotion called Lucky 7, where people can win free bottles of soda by looking under the cap. They advertise one in seven wins, and what they mean is that one out of every seven bottles has caps that say "winner" on them.

What's the probability that a person playing will win within his first three trials? Assume that this person also stops once he wins.

The three ways that he can win within the first three trials are:

- Win on the first trial
- Lose on the first trial, win on the second trial
- Lose on the first trial, lose on the second trial, and win on the third trial.

The only thing you don't want to have happened is for him to lose, lose, and lose. You can look at this in a tree diagram.



By looking at this tree diagram, you can see that the probability that he wins on the first trial, where x is the trial that he wins on, is 1/7. The probability that he wins on the second trial is 6/7 times 1/7 for this branch on the tree diagram. Then the probability that he wins on the third trial is 6/7 times 6/7 times 1/7.
To find the probability that he wins within the first three trials, you take all these values and add them together.

$$P(X=1)=\frac{1}{7}=0.143$$

$$P(X=2)=\left(\frac{6}{7}\right)\left(\frac{1}{7}\right)=0.122$$

$$P(X=3)=\left(\frac{6}{7}\right)^2\left(\frac{1}{7}\right)=0.105$$

$$\textit{Probability of winning within first three trial} =$$
$$P(X=1)+P(X=2)+P(X=3)=0.143+0.122+0.105=0.370$$

The probability that the person will win within the first three trials is 37%.

How do these calculations differ? Notice, every time there's a 1/7 representing the fact that he won. However, every subsequent value, there's another 6/7 fraction introduced into the calculation.

You can actually come up with a formula here to find the probability for the first success in a trial.

FORMULA TO KNOW

Geometric Distribution

$$P(X = k) = (1 - p)^{k-1}p$$

k *is the number of trials until first success*

p *is the probability of success*

If the geometric distribution is appropriate and x is the number of trials until you get a success, then the probability that it takes you exactly k trials to obtain a success probability of success is p. You only do that once.

$$P(x = k) \;=\; (1 - p)^{k-1}\,p$$

$(1 - p)^{k-1}$: *Fail every time until...*

p: *The last trial, when you succeed*

You fail every time except for the last time when you succeed.

TERM TO KNOW

Geometric Distribution

A probability distribution of the number of independent trials of a chance experiment it will take until a success. The criteria for a distribution to be geometric are (1) The chance experiment must only have two outcomes (success/failure) per trial, (2) the trials must be independent, (3) there must be a fixed probability of success for each trial, and (4) the variable of interest is the number of trials needed to obtain a "success" for the first time.

SUMMARY

Geometric probability follows the geometric setting. There are two outcomes per trial, success and failure. There is a fixed probability of success on each trial, and each trial has to be independent of each other. This time, instead of how many successes, you know you're only going to succeed once. You're interested in how many trials it takes to do that. When that's the case, the probability that it requires any particular number of trials can be found by multiplying the probability of failure together by all the trials except one of them, the last one, and multiplying by the probability of success the one time. Typically, these values are found on a calculator, although they can be found on a tree diagram.

Good luck!

TERMS TO KNOW

Geometric Distribution

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FORMULAS TO KNOW

Geometric Distribution

$P(X=k) = (1-p)^{k-1}p$

k is the number of trials until first success

p is the probability of success