Graph of a Logarithmic Function

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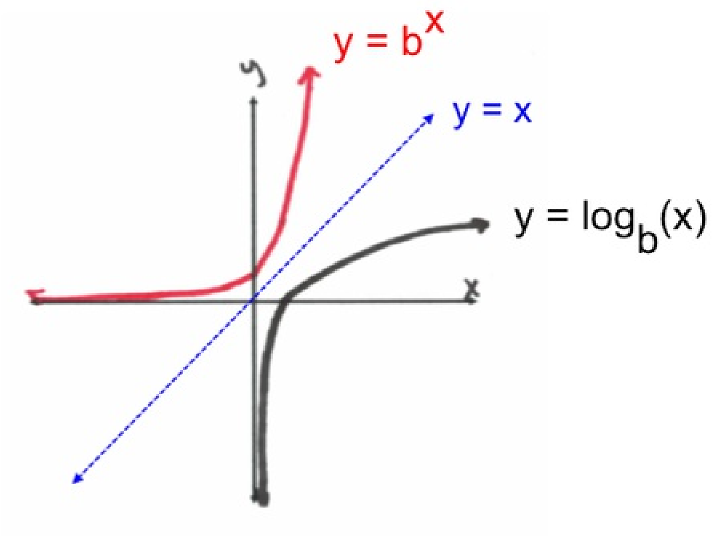
WHAT'S COVERED

In this lesson, you will learn to identify features of a logarithmic graph. Specifically, this lesson will cover:

1. Graph of Exponential Functions versus Graph of Logarithmic Functions

Exponents and logarithms are inverse operations. As functions, they are also inverses of each other. There is a special relationship between inverse functions on a graph: the line $y = x$ is a line of symmetry or reflection between a function and its inverse. So we should expect this relationship to hold true when examining the graphs of an exponential function and a logarithmic function:



2. Domain and Range of Logarithmic Functions

As we can see in the graph above, since a function is symmetrical about the line $y = x$ to its inverse, we can swap x- and y-values to plot points on a function's inverse. This also means that the domain and range of a function switches as we describe the domain and range of its inverse. That is, the domain of a function becomes the range of its inverse, and the range of a function becomes the domain of its inverse.

Looking at the graph of the exponential function, we can see that the domain is all x-values from negative infinity to positive infinity. This means that the range of logarithmic functions is all y-values, from negative infinity to positive infinity.

However, when looking at the range of the general exponential function graphed above, we see that the range is from zero to positive infinity. As the x-values approach negative infinity, the y-values approach 0 but never actually touch the x-axis. Also, there are no x values that make y negative (at least when both a and b are positive). So the range of an exponential function is y-values greater than zero, meaning the domain of the logarithmic function is restricted to x-values greater than zero through positive infinity. Inputting a negative value into the logarithmic function will yield a non-real answer.

BIG IDEA

The domain and range of a logarithmic function is the range and domain of an exponential function:

For exponential functions:

- The domain is all x-values.
- The range is restricted to y-values greater than zero.

For logarithmic functions:

- The domain is restricted to x-values greater than zero.
- The range is all y-values.

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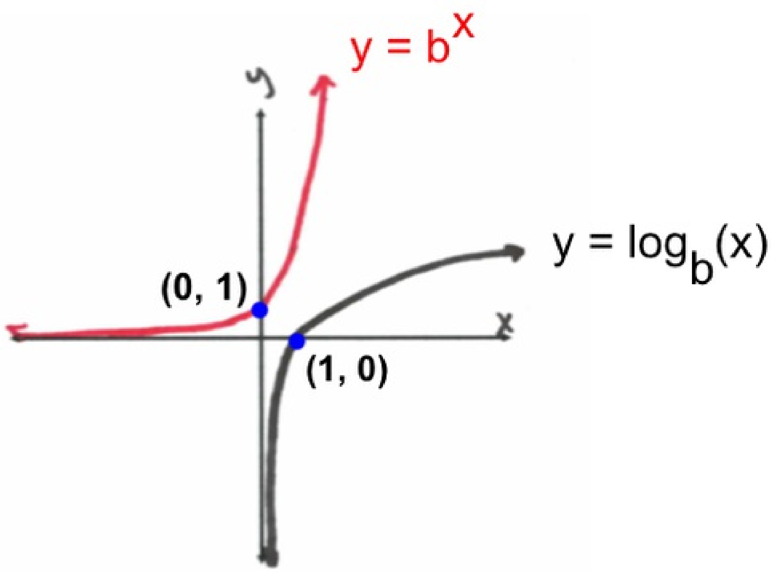
3. x-Intercept of Logarithmic Functions

When studying exponential functions, you may recall that the y-intercept of $y = b^x$ is at (0, 1) on the graph, because when x = 0, y = 1. This is because any value raised to the power of zero evaluates to 1. When comparing this to logarithmic functions, we invert x and y (maybe you are starting to see a pattern here!). So instead of talking about the y-intercept of an exponential function, we talk about the x-intercept of a logarithmic function; and instead of having coordinates of (0, 1), it has the coordinates (1, 0). This is illustrated in the graph below:



SUMMARY

When comparing the **graph of exponential functions versus the graph of logarithmic functions**, the graph of an exponential function reflected over the line *y* equals *x* is the graph of a logarithmic function. This is because exponential and logarithmic functions are inverses of each other. The **domain and range of logarithmic functions** are also inverses of the exponential functions. The domain of a logarithmic equation is all real *x* values greater than 0 and the range is all real *y* values. The **x-intercept of a logarithmic function** is (1, 0) and this is because 0 is equal to log of 1.