Sophia

# Graph of an Exponential Equation

*by Sophia*

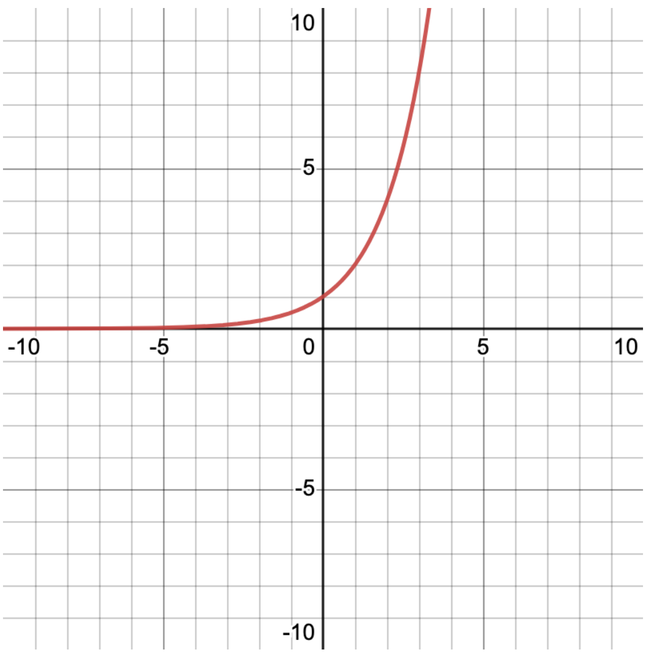
WHAT'S COVERED

In this lesson, you will learn how to identify features of an exponential graph. Specifically, this lesson will cover:

## 1. The Graph of $y = ab^x$

To learn more about the graph of the equation $y = ab^x$, let's look at a specific case.

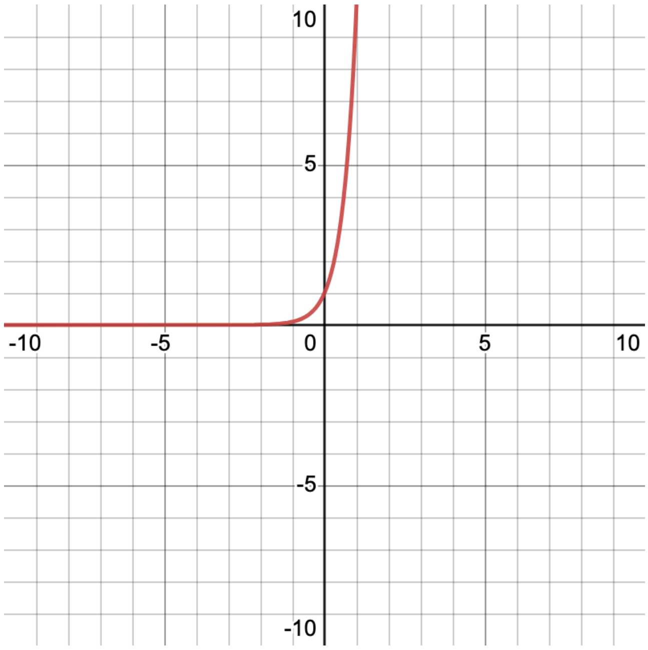
EXAMPLE Below is a sketch of the graph $y = 2^x$. Notice that the base *b* to the exponential expression is 2, and there is an implied scalar multiplier of 1 in front of this expression, which would be the *a* in the equation:




We see that, in this case, as $x$ gets larger (approaches positive infinity), the value of $y$ also increases (it approaches positive infinity as well). On the other hand, as $x$ gets smaller (approaches negative infinity), the value of $y$ gets smaller as well, but it approaches zero, rather than negative infinity.

This behavior is characteristic of exponential functions with a base larger than 1. If the base was between 0 and 1, then the behavior would be much different: $y$ would tend towards zero as $x$ gets larger, but tend toward infinity as $x$ gets smaller.

↪ EXAMPLE Consider the graph $y = 10^x$.




We have the same general behavior as in the previous graph, however, things are more dramatic: as $x$ becomes more positive, $y$ increases, but at a faster rate than in the previous graph. This is because a larger base number is being raised to a positive exponent. Similarly, as $x$ becomes more negative, $y$ decreases in value (approaching zero), but at a faster rate than in the previous graph. This is we can think of $y$ being divided by a larger number each time $x$ decreases in value.

HINT

Remember, the base can never be less than 0.

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# 2. The y-intercept of $y = b^x$ vs. $y = ab^x$

The y-intercept to any equation is the point on the graph at which the line or curve touches or crosses the y-axis. This always occurs when $x = 0$.

↪ EXAMPLE Let's return to the equation $y = 2^x$. When x equals 0, $y$ evaluates to 1, because any base number raised to a power of zero is 1. So we can say when $x$ equals 0, $y = 2^0 = 1$. That must mean this also holds true for the equation $y = 10^x$ from the second example above.

For the equation $y = b^x$, the y-intercept is at the point (0, 1) because when $x$ equals 0, $y$ equals 1.

But what about the y-intercept of equations in the form $y = ab^x$? We already know that $b^x$ evaluates to 1 when $x$ equals 0 for any base. We can deduce that the y-intercept depends on the value of $a$ in this case.

For the general exponential equation $y = ab^x$, the y-intercept has the coordinates $(0, a)$.

BIG IDEA

- For equations in the form $y = b^x$, the y-intercept has coordinates (0, 1).

 

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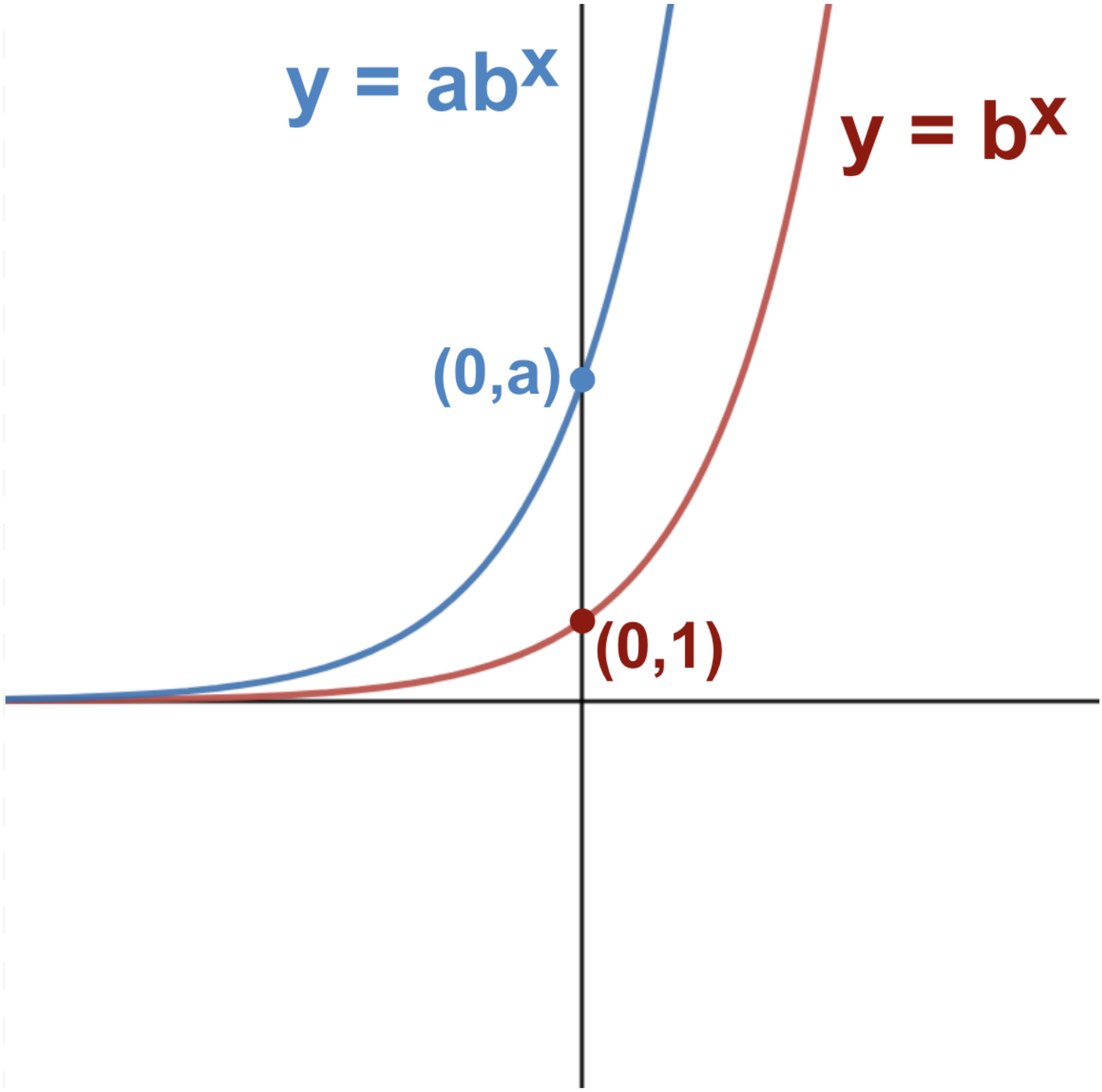



## 3. Features of the Graph of $y = ab^x$

The general exponential equation $y = ab^x$ has a positive $a$ and a positive *x*. If we reverse the signs of $a$ and *x* in the general exponential equation, we end up with different variations of the general exponential curve. More specifically, these are reflections about either the x- or y-axes or perhaps both, if the signs of both $a$ and *x* are reversed. These patterns are illustrated in the graphs below:

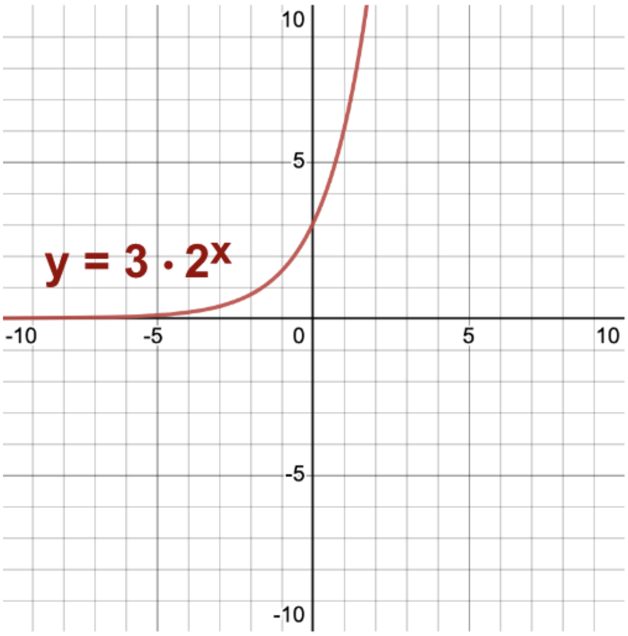
Let's look at the characteristics of each case.

### 3a. Positive *a*, Positive Exponent

Having a positive $a$ and a positive exponent is the general exponential equation:

$$y = ab^x$$

Suppose we have the equations $y = 3 \cdot 2^x$. In this graph, as $x$ is tending toward positive infinity, $y$ goes to positive infinity, and as $x$ goes to negative infinity, $y$ approaches 0.

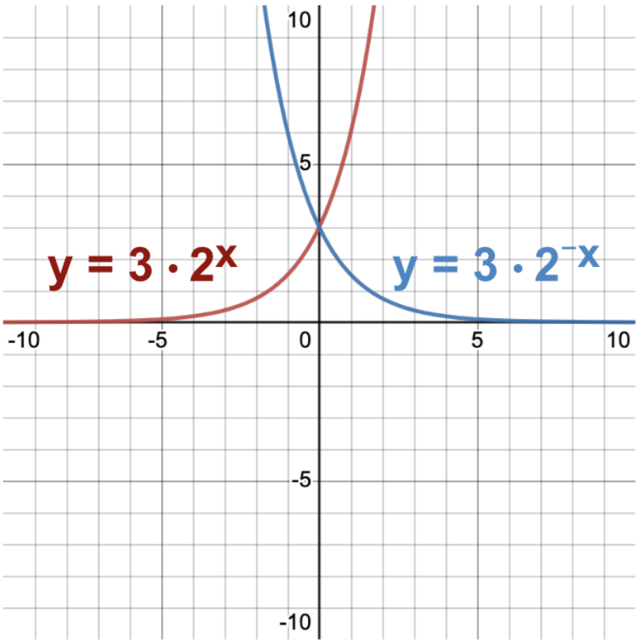



## 3b. Positive *a*, Negative Exponent

Let's take a further look into the next comparison of having a positive or negative exponent, but still having a positive $a$.

$$y = ab^{-x}$$

Suppose now we have the equation $y = 3 \cdot 2^{-x}$. A similar equation to $y = 3 \cdot 2^x$, but this equation has a negative exponent. We can see that since negative $x$ and positive $x$ are opposite, their graphs have opposite effects. When our exponent is negative, as $x$ approaches positive infinity, now $y$ is approaching 0, and as $x$ is approaching negative infinity, then our $y$ is approaching positive infinity.

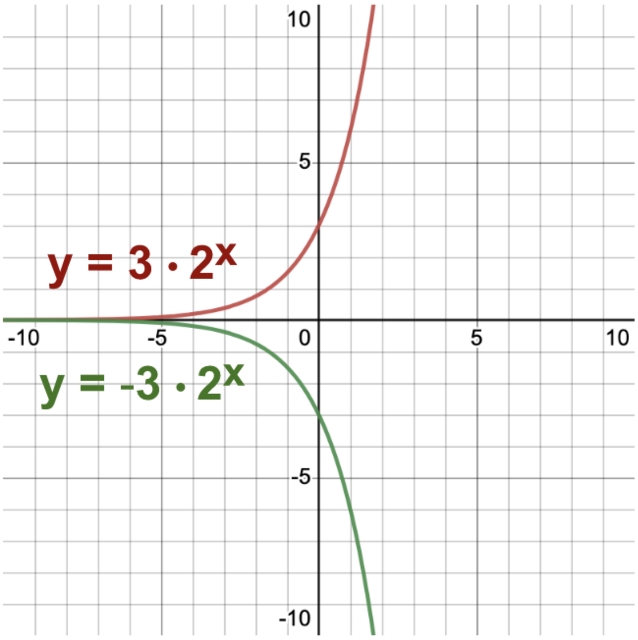



## 3c. Negative *a*, Positive Exponent

The next characteristic we'll look at is comparing the equations and graphs of exponential equations when the value of $a$, the number in front of the base, is positive or negative.

$$y = -ab^x$$

Suppose we still have our original exponential equation $y = 3 \cdot 2^x$. Let's compare that to the equation and graph of $y = -3 \cdot 2^x$. In the graphs, we can see that it looks like the graph is reflected over the x-axis. When the $a$ value is negative, as the x-values approach positive infinity, the y-values approach negative infinity. It's decreasing instead of increasing. Also, notice that as the x-values approach negative infinity, the y-values are still approaching 0 as they were with a graph of our equation with a positive $a$ value.

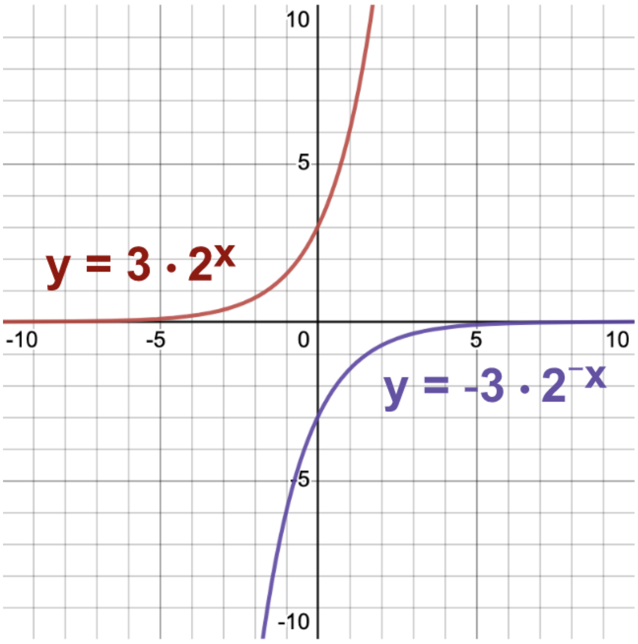



## 3d. Negative *a*, Negative Exponent

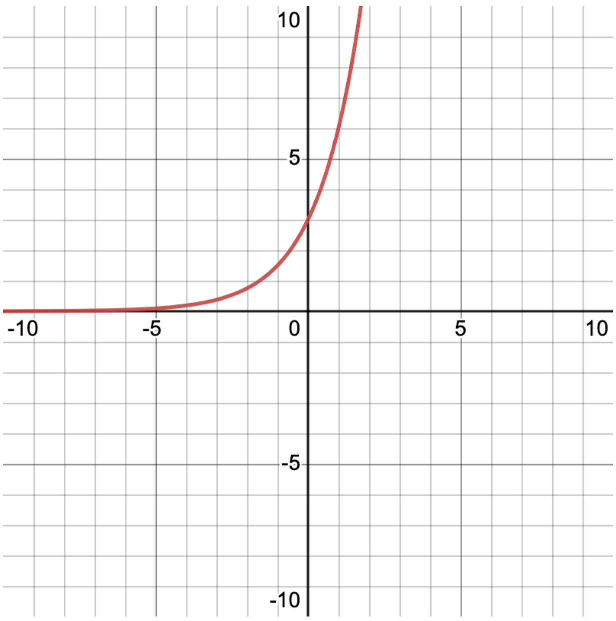
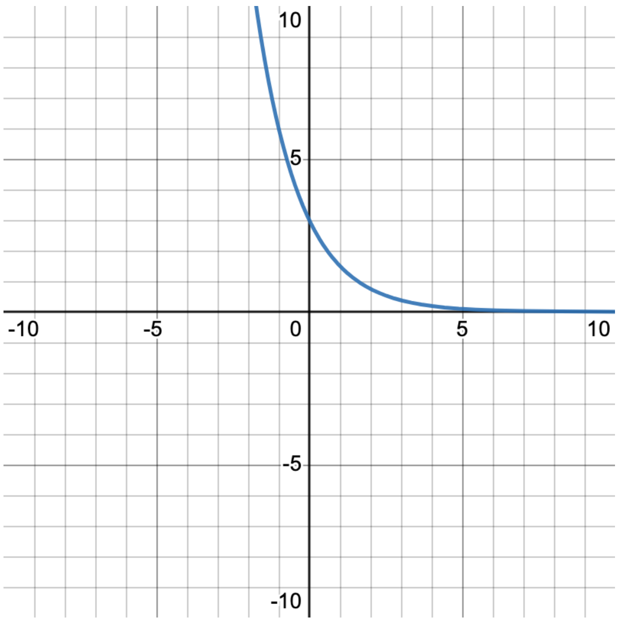
Finally let's look at the characteristics of exponential equations that have both a negative $a$ value and a negative exponent.

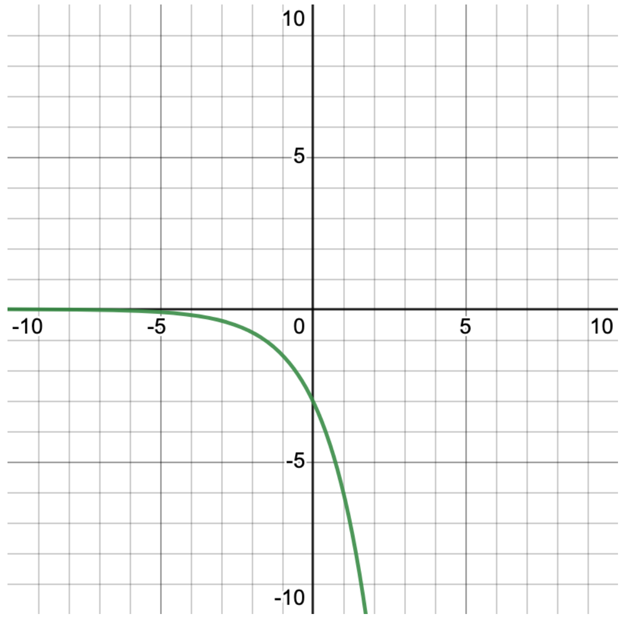
$$y = -ab^{-x}$$

Suppose now we have the equations $y = 3 \cdot 2^x$ and $y = -3 \cdot 2^{-x}$. This second equation has both a negative $a$ value and a negative exponent. When the equation has a negative $a$ and negative exponent, as the x-values approach positive infinity, we see that the y-values approach 0, and as the x-values approach negative infinity, we see that the y-values are also approaching negative infinity.

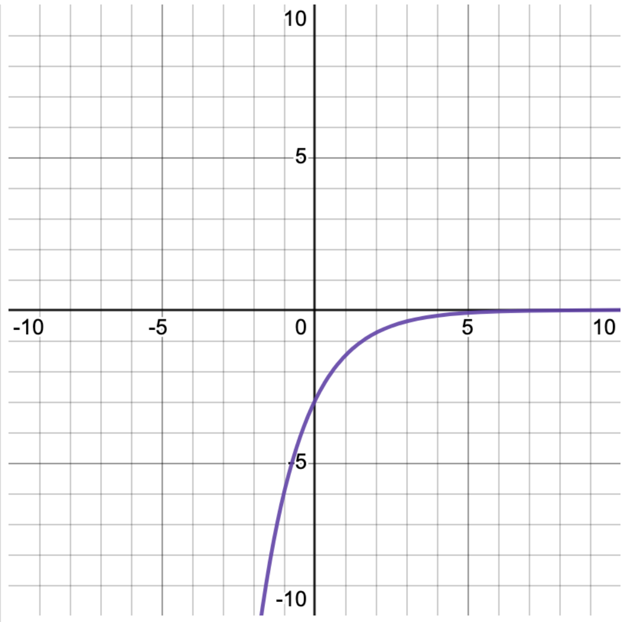



**BIG IDEA**

| $y = ab^x$ | $y = ab^{-x}$ |
|---|---|
|  |  |
| $x \to \infty, y \to \infty$<br>$x \to -\infty, y \to 0$ | $x \to \infty, y \to 0$<br>$x \to -\infty, y \to \infty$ |
| $y = -ab^x$ | $y = -ab^{-x}$ |
| | |



$x \to \infty, y \to -\infty$
$x \to -\infty, y \to 0$



$x \to \infty, y \to 0$
$x \to -\infty, y \to -\infty$

## SUMMARY

An equation in the form $ab^x$ is an exponential equation. The **graph of $b^x$** has certain characteristics. The **y-intercept of $ab^x$ function** is equal to the value of $a$ in the equation. Other features of the graph include looking at cases when $a$ is negative and when the exponent is negative. A negative exponent reflects the graph over the y-axis. while a negative $a$ coefficient reflects the graph over the x-axis.

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