Graphing Linear Equations

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WHAT'S COVERED

In this lesson, you will learn how to identify the corresponding graph of an equation in slope-intercept form or point-slope form. Specifically, this lesson will cover:

1. Using Slope-Intercept Form

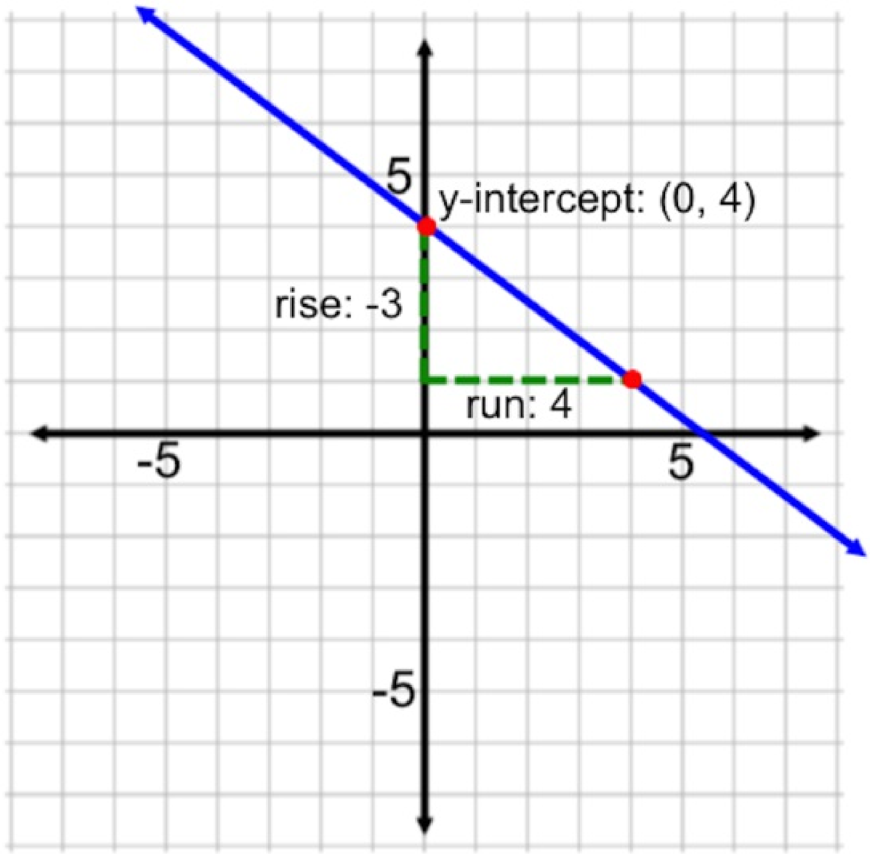
Slope-intercept form of a linear equation provides two useful pieces of information about the graph of the line: the slope of the line, and the location of the y-intercept. Slope-intercept form is $y = mx + b$, where *m* is the slope of the line, and *b* is the y-coordinate of the y-intercept. We know that all y-intercepts have an x-coordinate of 0, so the coordinate to the y-intercept is (0, *b*) in this form.

Recall that we only need two points in order to graph a line. Once we have two points plotted, we can connect the two points and extend the line in both directions.

- When graphing a line using its equation in slope-intercept form, plot the y-intercept first. This gives us one of our two points needed.
- To find the other point, we'll use the slope. The slope is the ratio between rise and run, or vertical distance over horizontal distance from one point to another. Starting at the y-intercept, the slope will tell us how to move vertically and horizontally to another point on the line. We won't have to go far until we find another point on our line.

EXAMPLE Graph the line $y = -\frac{3}{4}x + 4$.

First, plot the y-intercept, which will be (0, 4). Then use the information from the slope to plot the second point. The slope in this case is $-\frac{3}{4}$. This tells us we have a rise of -3 and a run of 4. So starting at (0, 4), go down 3 and over to the right 4. We have another point at (4, 1). Plot this second point and connect the two points with a line.



BIG IDEA

To graph a line given its equation in slope-intercept form, plot the y-intercept first, using the b-value in the equation. To find a second point on the line, use the slope, *m*, to move vertically and horizontally from the y-intercept. Finally, connect the two points and extend them in both directions to plot the line.

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2. Using Point-Slope Form

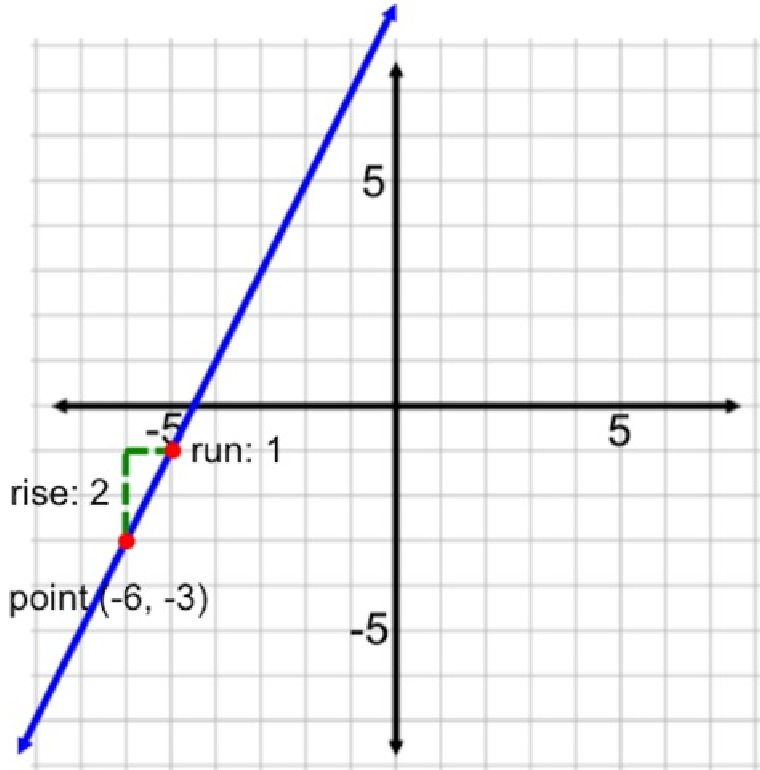
When given the equation of a line in point-slope form, we follow a similar process as above: we can easily plot one point on the line from the equation. From there, we can use the slope to move vertically and horizontally to another point on the line, and then draw the line on the graph.

EXAMPLE Graph the line $y+3=2(x+6)$.

First, identify the given point. Recall that the point-slope form is $y-y_1=m(x-x_1)$, where *m* is the slope and (x_1, y_1) is a point on the line. From the equation $y+3=2(x+6)$, we can identify the slope as $m=2$.

To find the point, be careful with positives and negatives when working in this form. The general equation is $y-y_1=m(x-x_1)$. This means that if you see a positive value after *y* or *x*, the coordinate is actually a negative value. In the above equation, we have $y+3$, which means the y-coordinate is actually -3. We also have $(x+6)$ which means the x-coordinate is actually -6. The point included on this line is $(x_1, y_1)=(-6, -3)$.

Plot the point (-6, -3). A slope of 2 is the same as a slope of $\frac{2}{1}$. So the rise is 2 and the run is 1. From the point (-6, -3), go up 2 and over to the right 1 to get a second point at (-5, -1). Plot this second point and connect the two points with a line.



HINT

As stated above, be cautious of positives and negatives in your equation when written in point-slope form! If we had the equation $y-4=2(x+5)$, the point would be (-5,4). Here are a few more examples:

Point-Slope Form	Point	Slope
$y-2=12(x-5)$	(5,2)	12
$y+9=-3(x-7)$	(7,-9)	-3
$y-1=\frac{1}{4}(x+6)$	(-6,1)	$\frac{1}{4}$
$y+5=-\frac{2}{3}(x+4)$	(-4,-5)	$-\frac{2}{3}$

BIG IDEA

When given an equation in point-slope form, use the x- and y-coordinates from the equation to plot one point on the line. Next, use the information about slope to move vertically and horizontally from one point to find the location of another point. Once you have two points plotted, connect them and extend the line to complete the graph.

3. Using Standard Form

While the other two forms instantly provide information about the slope of a line, and either the y-intercept or some other point on the graph, equations written in standard form can seem unhelpful at first. However, standard form allows us to easily find the line's x- and y-intercepts. Recall that at our intercepts, either x or y will have a value of zero; and because we have both the x-term and the y-term in the equation, when one of x or y is zero, the entire term has a value of zero. This makes calculating intercepts rather easy, and if we can easily find both intercepts, we can easily graph the line.

→ EXAMPLE Graph the line $3x+2y=12$.

Recall that the x-intercept has the coordinate (x, 0). To find the x-intercept, plug in zero for y and solve for x.

$3x+2y=12$	Using the standard form equation, plug 0 in for y and solve for x
$3x+2(0)=12$	Evaluate 2 times 0
$3x=12$	Divide both sides by 3
$x=4$	Use this value to find the coordinates of the x-intercept
$(4, 0)$	Coordinate of x-intercept

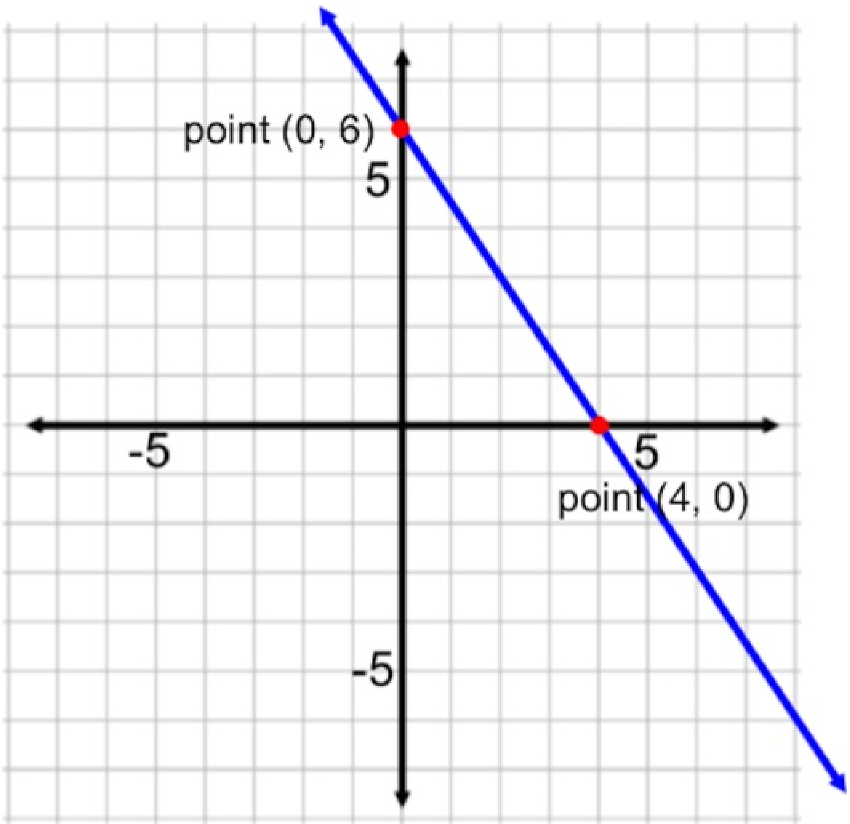
The x-intercept is at the point (4, 0).

Similarly, the y-intercept has the coordinate (0, y). Plug in zero for x and solve for y to find the y-intercept.

$3x+2y=12$	Using the standard form equation, plug 0 in for x and solve for y
$3(0)+2y=12$	Evaluate 3 times 0
$2y=12$	Divide both sides by 2
$y=6$	Use this value to find the coordinates of the y-intercept
$(0, 6)$	Coordinate of y-intercept

The y-intercept is at the point (0, 6).

Now that we have two points for our line, we can plot them and graph the line.



BIG IDEA

Standard form is ideal for finding both the x- and y-intercepts to a line. Plug in 0 for *y* and solve for *x* in order to find the x-intercept, and plug in 0 for *x* and solve for *y* to find the y-intercept. Once both intercepts are found, we can plot them on the graph and complete the graph of the line.

SUMMARY

To graph a line, we only need to know the coordinates of two points on the line. **Using slope-intercept form**, a line can be determined with the variable *b*, which is the y-coordinate of the y-intercept, and the variable *m*, which is the slope. **Using point-slope form**, a line can be determined with the point in the equation and the variable *m*, which is the slope. Finally, **using standard form** allows us to easily find the x- and y-intercept.

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