Identifying the Graph of a Quadratic Equation

by Sophia

WHAT'S COVERED

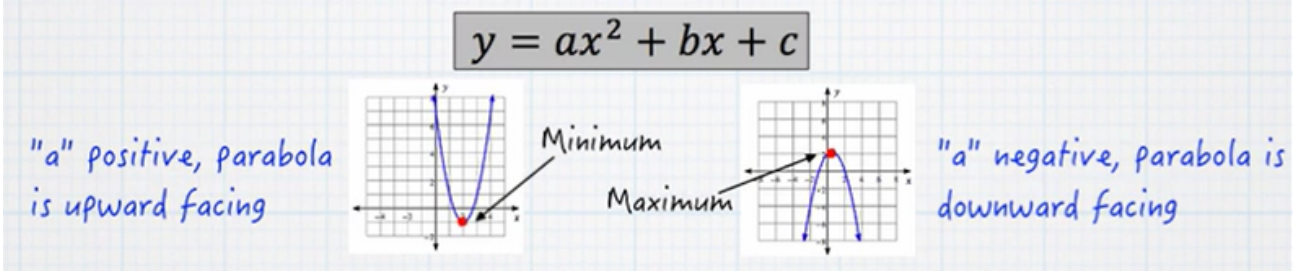
This tutorial covers how to identify the graph of a quadratic equation, also known as a parabola, through the discussion of:

1. Parabolas: A Review

In review, the graph of a quadratic equation is called the parabola, which can either open upwards or downwards. Parabolas have a vertex, which represents a minimum or maximum point on the graph depending on the direction of the parabola.

Quadratic equations in the form below can provide information about the shape and vertex of the parabola:

- If the value of the coefficient *a* in the equation is positive, then the parabola opens upwards and the vertex represents a minimum point.
- If the value of the coefficient *a* in the equation is negative, then the parabola opens downwards and the vertex represents a maximum point.



2. Calculating the Vertex of a Parabola

Equations in the quadratic form below can also help you determine the coordinate of the vertex of the parabola.

$y = ax^2 + bx + c$

The values of *a* and *b* and **the vertex formula** below are used to find the x-coordinate of the vertex of the parabola.

FORMULA TO KNOW

x-coordinate of Vertex

$$x = \frac{-b}{2a}$$

Once you have the x-coordinate of the vertex of the parabola, you can then substitute this value into the quadratic equation to find the y-coordinate of the vertex.

3. Determining the x- and y-Intercepts of a Parabola

Given what you've learned so far, how can you find the intercepts of a parabola? The y-intercept is the point at which the graph intersects the y-axis, and on the y-axis the value of *x* is always 0. Therefore, you can determine the y-intercept by substituting 0 for *x* in the equation and solving for *y*. In the general equation below, when *x* equals 0, the x^2 term will equal 0, and the *x* term will equal 0.

$y = \cancel{ax^2} + \cancel{bx} + c$

Therefore, the y-intercept of any quadratic equation is given by the constant term c.

The x-intercepts are the points at which the graph intercepts the x-axis. On the x-axis, the value of *y* is always 0. Therefore, you can determine the x-intercepts by substituting 0 for *y* in the equation and solving for *x*.

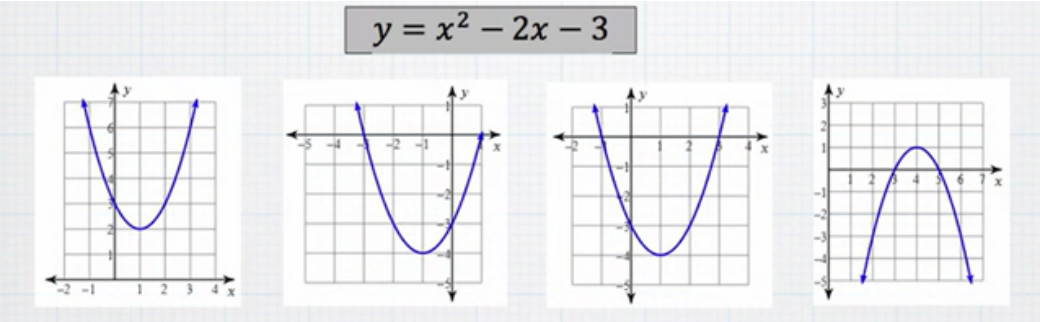
DID YOU KNOW

Finding the x-intercepts of any parabola is the same as solving the quadratic equation, since the solutions to a quadratic equation are found by setting the equation equal to 0. Therefore, the x-intercepts of a parabola are also known as the roots, 0s, or solutions of the quadratic equation.

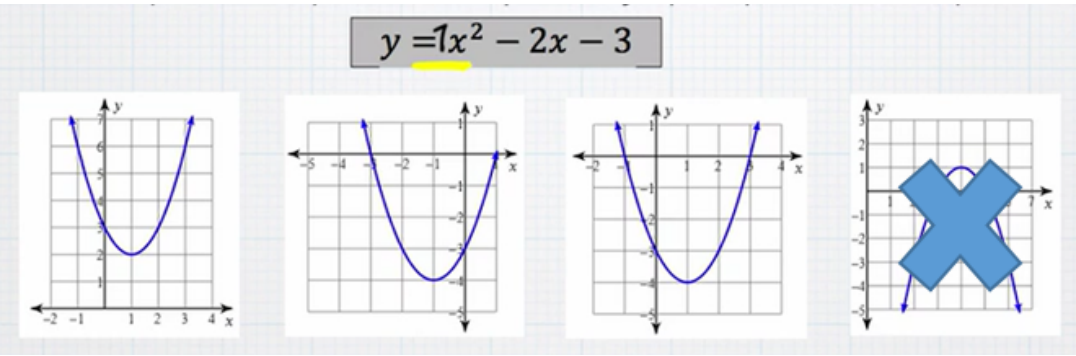
4. Identifying a Graph from an Equation of a Parabola

Given a quadratic equation, you can use information about the sign of the coefficient *a*, the vertex, and the x- and y-intercepts to identify its corresponding graph.

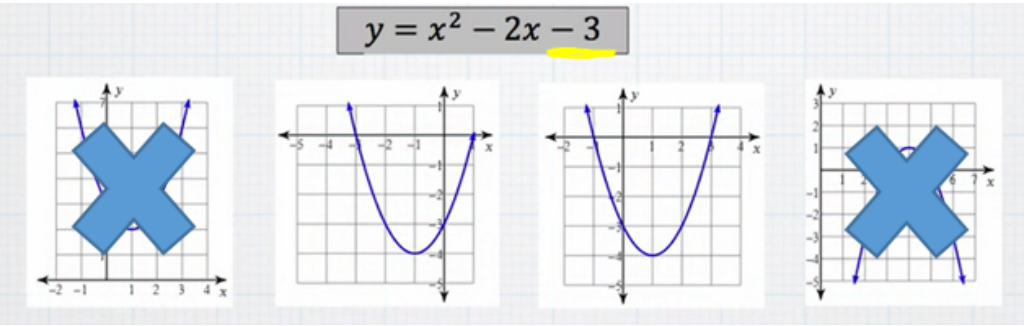
EXAMPLE Given the quadratic equation below, which of these four graphs represents the equation?



First, you can see that the value of *a* is 1, which is positive. Therefore, you know that the graph should be facing upwards, so you can eliminate the last graph.



Also, you can see that the value of c is -3, which means that the y-intercept of the graph is -3. Therefore, you can eliminate the first graph.

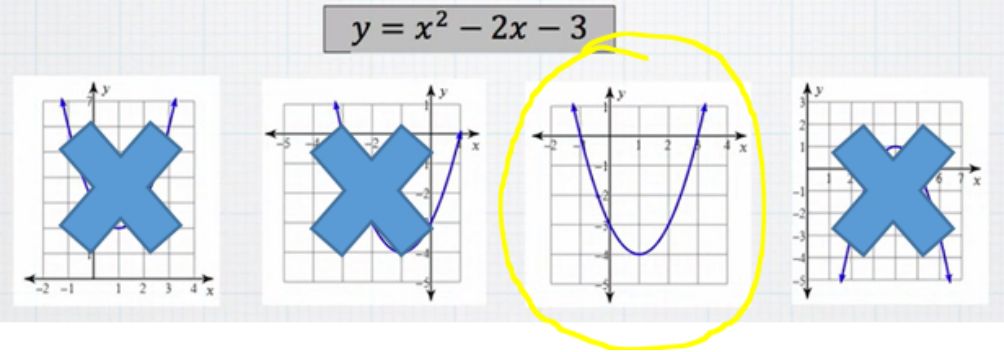


Next, you can use the formula for the x-coordinate of the vertex (the vertex formula) to find the x-value of the

vertex. Substitute 1 for *a* and -2 for *b* and simplify:

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$$

Since your solution is *x* equals 1, then you know that the x-coordinate of your vertex is a positive 1. Therefore, you can eliminate the second graph, which has an x-coordinate of -1. This means that the remaining graph, the third graph, matches your equation. The x-intercepts are at (-1, 0) and (3, 0). These are the roots or 0s of the equation, or the x-values, that make *y* equal to 0.



SUMMARY

Today you reviewed **parabolas** and **calculating the vertex of a parabola** using the vertex formula. You learned **how to determine the x- and y-intercepts of a parabola**, noting that 1) the y-intercept of a graph is when the value of *x* is equal to 0, and 2) the y-intercept of any quadratic equation is given by the constant term c. You also learned that an x-intercept of a graph occurs when the value of *y* is equal to 0; therefore, finding the x-intercept of a parabola is the same as solving the quadratic equation, since the solutions to a quadratic equation are found by setting the equation equal to 0. This is why the x-intercepts of a parabola are also known as the roots, 0s, or solutions to a quadratic equation. You learned that you can use this information about the x- and y-intercepts, along with information about the equation's coefficient and vertex, to **identify a graph from an equation of a parabola**.

Source: This work is adapted from Sophia author Colleen Atakpu.

FORMULAS TO KNOW

x-coordinate of Vertex

$$x = \frac{-b}{2a}$$