Imaginary Numbers

by Sophia

WHAT'S COVERED

This tutorial covers imaginary numbers, through the definition and discussion of:

1. Squaring and Square Roots: A Review

The square root of a number *x* is the number whose product with itself is *x*.

EXAMPLE If you square the number -2, it equals 4. If you square the number 2, it also equals 4.

$(-2)^2 = 4 \qquad 2^2 = 4$

As you can see from the examples above, when you square any real number, the result will *never* be a negative number.

2. Imaginary Numbers

Since the square of a real number cannot be negative, the square root of a negative number must be a non-real number, otherwise known as an imaginary number. The **imaginary unit**, *i*, is defined as the square root of -1.

FORMULA TO KNOW

Imaginary Number
$\sqrt{-1} = i$

Imaginary Unit
The square root of -1, denoted by i

Imaginary numbers may be the result of solving a quadratic equation using **the quadratic formula**.

FORMULA TO KNOW

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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3. Writing Imaginary Numbers

Imaginary numbers are written using the imaginary unit *i* in the form *bi* (b times i), where b is a real number. Recall that the product property for square roots states that for positive numbers a and b, the square root of a times b is equal to the square root of a times the square root of b

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

You can also use the product property for square roots of negative numbers in the form *bi*.

EXAMPLE The square root of -9 can be written as the square root of 9 times the square root of -1. The square root of 9 is 3, and the square root of -1 is defined as *i*. Therefore, you can write the square root of -9 as 3*i*.

$$\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1} = 3i$$

Being able to identify perfect squares and appropriately using the product property for square roots is important when you are simplifying square roots and writing imaginary numbers.

EXAMPLE Suppose you want to simplify the expression:

$$\sqrt{12 - (7-3)^2}$$

You can start by simplifying in your parentheses.

$$\sqrt{12 - 4^2}$$

Next, you square the 4, and subtract your terms, which equals -4.

$$\sqrt{12 - 16} = \sqrt{-4}$$

Using the product property for square roots, you can rewrite the square root of -4 as the square root of 4 times the square root of -1. The square root of 4 is 2, and the square root of -1 is i, so your final answer is 2*i*.

$$\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1} = 2 \cdot i = 2i$$

TRY IT

Consider the following expression:

$$\sqrt{4 \cdot 3 - 15}$$

Simplify this expression. +

First, simplify underneath the square root, starting with multiplication, followed by subtraction.

$$\sqrt{12 - 15} = \sqrt{-3}$$

Now you can rewrite your expression using the product property of square roots. Note that since the square root of 3 is not an integer, you would leave it as the square root of 3. Since the square root of -1 is *i*, your final answer is the square root of 3 times *i*.

$$\sqrt{-3} = \sqrt{3} \cdot \sqrt{-1} = \sqrt{3} \cdot i = \sqrt{3}i$$

SUMMARY

Today you reviewed **squaring and square roots**, recalling that the square root of a number x is the number whose product with itself is x. Remember, the square of any real number will never be a negative number, and the square root of a negative number must be a non-real or **imaginary number**. You learned that this imaginary unit *i* is defined as the square root of -1. Lastly, you learned that when **writing imaginary numbers**, you use the imaginary unit, *i*, in the form *bi*, where b is a real number.

Source: This work is adapted from Sophia author Colleen Atakpu.

TERMS TO KNOW

Imaginary Unit

The square root of -1, denoted by i.

FORMULAS TO KNOW

Imaginary Number

$$i = \sqrt{-1}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$