

Independent vs. Dependent Events

by Sophia



WHAT'S COVERED

This tutorial will discuss the difference between events that we can consider independent versus dependent events. Our discussion breaks down as follows:

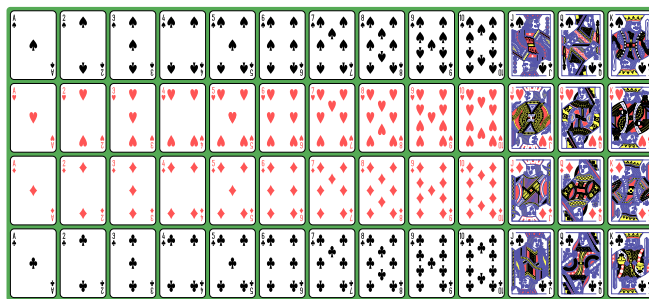
1. Independent Events
2. Sampling
 - 2a. With Replacement
 - 2b. Without Replacement
3. Dependent Events

1. Independent Events

There is a difference between events that are independent of each other and ones that we can't consider independently of each other. Once you define one, you can figure out the other one.

Independent events refer to scenarios in which knowing the outcome of the first event doesn't affect the probabilities for the second event.

⇒ **EXAMPLE** Suppose that you did an experiment that consisted of two parts: selecting a card from the 52 cards in a deck and rolling a die.



+



You don't know what card you're going to pick and you don't know what number you're going to get on a die.

What if you have an additional piece of information? What if you knew that the card that you were going to pick was the four of clubs? Would that provide any additional information about what the die was going to be when you rolled it? No, it wouldn't, and so we can call these events independent.

Other examples of independent events include:

- *Rolling three dice:* Knowing what happened with the first die doesn't affect what's going to occur on the second die, or even third die.
- *Flipping two coins:* Knowing that one coin came up heads doesn't affect the probability that the second one will come up heads or tails.



TERM TO KNOW

Independent Events

Two events where knowing whether the first event occurred does not affect the probability of the second event occurring.

2. Sampling

Certain examples need a caveat to be considered independent based on whether or not they are sampling with replacement or sampling without replacement.

2a. With Replacement

Sampling with replacement is a sampling method where each selected item is replaced into the sampling frame before the next trial. Using this method, an item can be selected more than once.

↪ **EXAMPLE** Selecting two marbles from a jar or drawing two cards from a deck can only to be considered independent if you replace the first marble or card before you select the second item.

You have to put the first marble back and mix up the marbles in the jar before you select the second one, or you have to put the first card back and shuffle the deck before you select the second one.

If the first marble or card is replaced, you can consider the two draws from the jar or two draws from the deck to be independent. The probabilities of each draw won't change.



TERM TO KNOW

Sampling with Replacement

A sampling method where each selected item is replaced into the sampling frame before the next trial. Using this method, an item can be selected more than once.

2b. Without Replacement

Sampling without replacement is a sampling method where each selected item is not replaced into the sampling frame before the next trial. Using this method, an item can only be selected once, and the probabilities of particular events may change as subsequent trials are performed.



THINK ABOUT IT

What is the probability that you will draw a spade from the deck of cards? The probability that you get a spade on the first draw is one-fourth because one out of every four cards are spades.

Suppose that you draw a spade the first time and do not replace it. That would be considered sampling without replacement.

If the two events of drawing were independent, then the probability of selecting a spade wouldn't have changed for the second draw. However, now only 51 cards remain because you removed one of the spades, leaving 12 spades left. 12 spades out of 51 cards is not one out of four. Therefore, the probability of drawing a spade is not the same as it was on the first draw.



TERM TO KNOW

Sampling Without Replacement

A sampling method where each selected item is not replaced into the sampling frame before the next trial. Using this method, an item can only be selected once, and the probabilities of particular events may change as subsequent trials are performed.

3. Dependent Events

In the previous case of sampling without replacement, knowing that you got a spade on the first draw will affect the probability of a spade on the second draw. Thus, these are **dependent events**.



BIG IDEA

With dependent events, knowing what happened on the first event affects the probability for the second event.

Let's explore another scenario.

⇒ **EXAMPLE** Suppose you roll a die but do not look at the face. What is the probability that you will have rolled an odd number? Well, three of the die's faces are odd, so your probability is $\frac{3}{6}$, or $\frac{1}{2}$. Now suppose that you are told that the number you rolled is a high number, and you know that 1, 2, and 3 are considered low, and 4, 5, and 6 are considered high. What's the probability now of your roll being odd?

You know you're limited to the high numbers, 4, 5, and 6, of which only 5 is odd. The probability of an odd in that selection is $\frac{1}{3}$. Knowing that it was high changed the probability of it being an odd number.

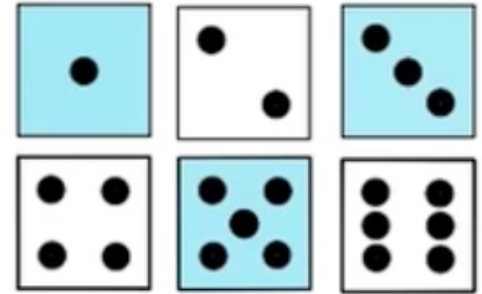
When you add this new layer of knowledge, the probability gets affected, which means that the events in question--high number and odd number--are dependent on a die.



TERM TO KNOW

Dependent Events

Two events where knowing whether the first event occurred affects the probability of the second event occurring.



SUMMARY

Independent events are events where the knowledge of what happened on the first trial doesn't change the probability that the second event will occur. Many times, independent events occur when sampling with replacement, whereas dependent events occur if you sample without replacement.

Good luck!

Source: THIS TUTORIAL WAS AUTHORED BY JONATHAN OSTERS FOR SOPHIA LEARNING. PLEASE SEE OUR [TERMS OF USE](#).



TERMS TO KNOW

Dependent Events

Two events where knowing whether the first event occurred affects the probability of the second event occurring.

Independent Events

Two events where knowing whether the first event occurred does not affect the probability of the second event occurring.

Sampling with replacement

A sampling method where each selected item is replaced into the sampling frame before the next trial. Using this method, an item can be selected more than once.

Sampling without replacement

A sampling method where each selected item is not replaced into the sampling frame before the next trial. Using this method, an item can only be selected once, and the probabilities of particular events may change as subsequent trials are performed.