Introduction to a System of Inequalities

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to identify a solution for a system of inequalities on a graph. Specifically, this lesson will cover:

1. Systems of Inequalities vs Systems of Equations

It should go without saying that the main difference between a system of inequalities and a system of equations is that a **system of inequalities** consists of at least two inequalities, rather than two equations.

As we will discuss below, there is also a major difference in how we interpret the solutions to a system of inequalities. However, one thing remains the same: the variables within the inequalities of the system must have matching definitions. Otherwise, we cannot consider the relationship to be a system. This becomes important when working with systems of inequalities in a situational context.

EXAMPLE If x represents apples in one inequality, it must represent apples in all of the inequalities in the system.

TERM TO KNOW

System of Inequalities
Two or more inequalities with the same variables, considered at the same time.

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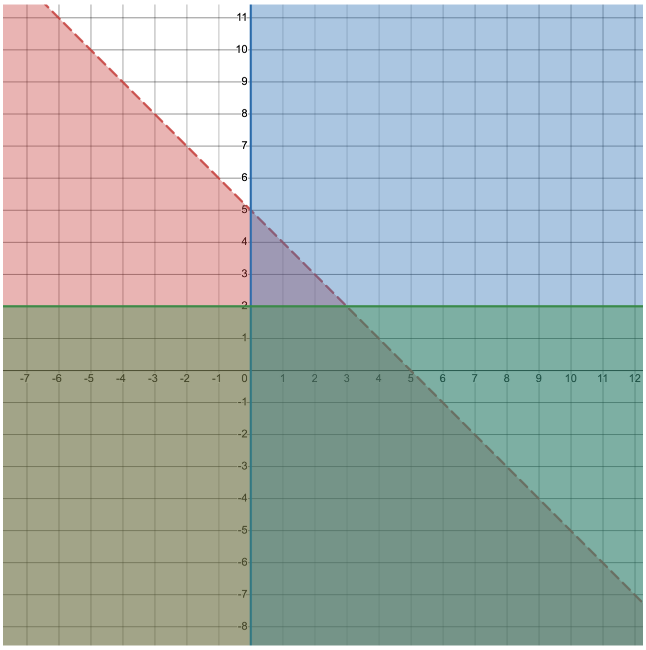
2. The Solution Set of a System of Inequalities

Just as a solution to a system of equations satisfies all equations in the system, solutions to a system of inequalities must satisfy all inequalities in the system. Sometimes, a coordinate pair (x, y) will satisfy one or two of the inequalities in the system, but not all. In these cases, the coordinate pair does not represent a solution to the entire system. One way to think about inequalities is that they represent boundaries. In a system, there are several boundaries, each of which is represented by an inequality. In this sense, a solution to a system of inequalities fits with every boundary defined by the inequalities.

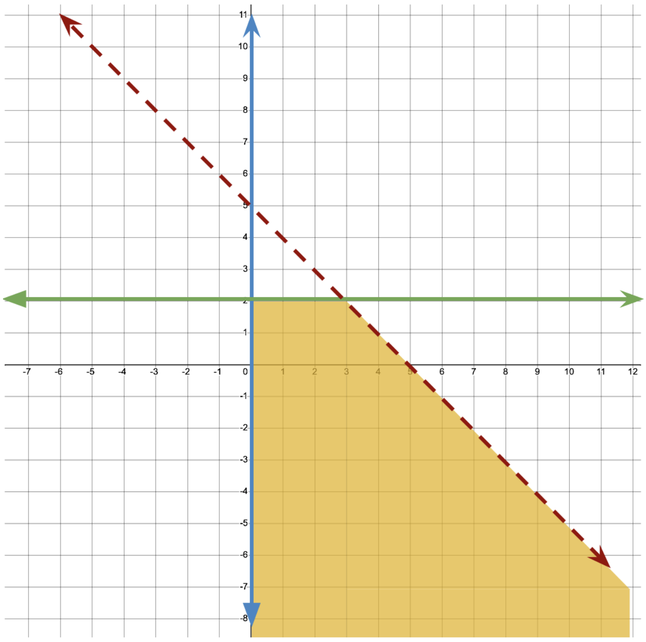
Let's take a visual approach to understand solutions to a system of inequalities.

↱ EXAMPLE Below is a graph of the following system:

$$x + y < 5$$
$$x \geq 0$$
$$y \leq 2$$



It can be a bit tough to see but there is a solution area where all three regions overlap. Here is a graph with just the solution area.



The region highlighted in yellow is referred to as the solution region of the system. All coordinate points (*x*, *y*) that exist within this solution region satisfy all of the inequalities. In other words, they fit within all boundaries defined by the system.

Let's test a few points to see if they are a solution to this system:

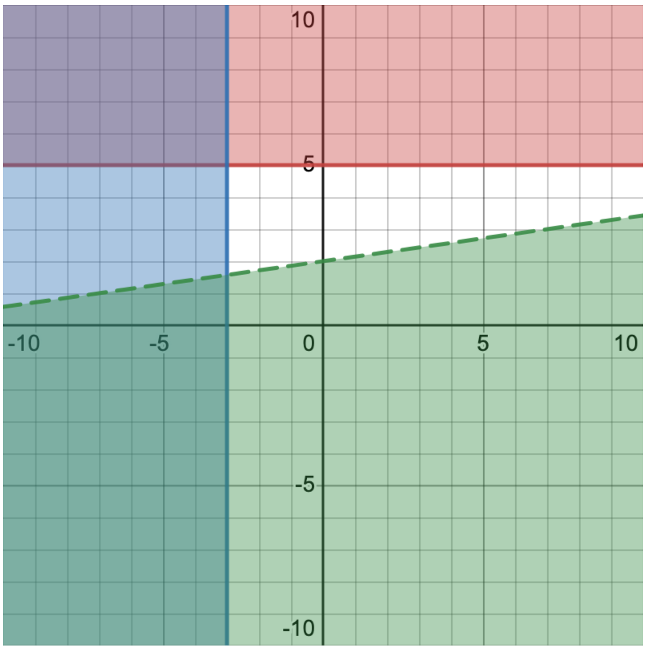
Point	Solution?	Explanation
(2, 1)	Yes	(2, 1) is within the shaded region.
(-2, 2)	No	Although (-2, 2) lies on the green line, it is outside the shaded region.
(0, 5)	No	Although (0, 5) lies on both the red and blue line, it is outside the shaded region.
(6, -4)	Yes	(6, -4) is within the shaded region.
(8, 6)	No	(8, 6) is outside the shaded region.

3. Systems of Inequalities with No Solution

Sometimes a graph of a system will have overlapping solution regions to individual inequalities in the system, without an overlap of solutions to all inequalities. Recall that solutions to the entire system must fit within all boundaries defined by the system. Thus, when no overlap between all inequalities exist, there is no solution to the system, although we may be tempted to think there are, because some overlaps exist.

→ EXAMPLE Below is a graph of the following system:

$$y \geq 5$$
$$x \leq -3$$
$$y < \frac{1}{7}x + 2$$



This system actually has no solution. This is because there is no coordinate pair (x, y) that fits within all the boundaries of the system. We can identify points that will satisfy two of the three inequalities. For instance, there is an overlap between the blue and red regions and an overlap between the blue and red regions. There is even an overlap between the red and green regions that we just can't see on the graph as it is. However, at no point will there be an overlap between all three regions. For this reason, there is no solution to this system.

SUMMARY

When looking at **systems of inequalities vs systems of equations**, the big difference is that a system of inequalities consists of at least two inequalities, rather than two equations. A system of linear inequalities can be used to represent a situation with several boundaries. **The solution set of a system of inequalities** fits within all boundaries defined by the system. This is shown on the graph as the region that is overlapped by all shaded region. Any point within this region represents a solution to the system. There are some **systems of inequalities with no solution** will have overlapping solution regions to individual inequalities in the system without an overlap of solutions to all inequalities.

TERMS TO KNOW

System of Inequalities
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