Introduction to Arithmetic Sequences

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WHAT'S COVERED

In this lesson, you will learn how to calculate the value of the nth term or the term number that has a specific value in an arithmetic sequence. Specifically, this lesson will cover:

1. Sequences

In math, a sequence is a set of numbers in a particular order. For example, {1, 3, 5, 7, 9} is a sequence, because we have numbers in a set (denoted by the curly braces) and the numbers are in numerical order. This particular sequence is finite because there are a limited number of **terms** (there are only 5 numbers).

Sequences can be infinite, meaning that they are an endless number of terms. {1, 3, 5, 7, 9...} is an infinite sequence, denoted by the ellipsis (the "dot-dot-dot"). This means that the sequence continues to include all positive odd integers.

TERM TO KNOW

Term (in a Sequence)
Refers to the place or order of a number in a sequence, such as first, second, third, etc.

2. Arithmetic Sequences

The example sequence above also happens to be an example of an **arithmetic sequence**. Like all sequences, it is a set of numbers in numerical order. What makes it an arithmetic sequence is the constant change in value from one term to the next.

EXAMPLE In the sequence {1, 3, 5, 7, 9...}, we add 2 to each term to get the value of the term after it. This constant change in value is called the **common difference** between terms.

THINK ABOUT IT

Now that we know that {1, 3, 5, 7, 9...} is an arithmetic sequence with a common difference of 2, what are the values of the next two terms in the sequence? We just continue to add 2 to the last term we know. The value of the next two terms are 11, and 13.

TERMS TO KNOW

Arithmetic Sequence
A set of numbers in numerical order, with a common difference between each term.

Common Difference
The numerical distance between any two consecutive terms in an arithmetic sequence, a constant value.

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3. The Formula for Finding a Term in an Arithmetic Sequence

It was easy enough to find the value of the next two terms in the sequence above because we could just add 2 a few times. What if we wanted to find the value of the 50th term? Or the 400th term? Certainly adding 2 by hand hundreds of times isn't the easiest way. Instead, we can use this formula:

FORMULA TO KNOW

Arithmetic Sequence
$a_n = a_1 + d(n-1)$

In this formula, we can define each variable as:

- a_n: the value of the nth term
- a_1: the value of the 1st term
- d: the common difference
- n: the term

Next, we are going to use this formula to write formulas for specific arithmetic sequences, and then solve for the value of the nth term, as well as find n given its value.

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4. Writing a Formula for an Arithmetic Sequence

To write the formula for an arithmetic sequence, we need to identify variables that are available with the given information.

EXAMPLE Consider this sequence: {7, 11, 15, 19, 23, 27, 31, ... }

How can we write the formula to describe the value of any term in this sequence?

We know that part of the formula is the value of the first term, so we know that 7 will be included in the formula.

The biggest thing is to find the common difference. Remember that the common difference is the numerical distance between any two consecutive terms in an arithmetic sequence. So just pick two terms that are next to each other, and subtract one from the other. Let's choose the terms 23 and 19. The difference between these two numbers is: 23 – 19, or 4.

So we have our common difference of 4, and the initial term is 7. Let's put this into our formula:

$a_n = 7 + 4(n-1)$

HINT

It is a good idea to check that the common difference is 4 throughout the sequence (in case what we are working with isn't arithmetic at all! Also, keep in mind that the common difference may be negative; this is the case when the terms are decreasing in value rather than increasing.

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5. Use the Formula to Find a_n and n

Now that we have a formula to describe the arithmetic sequence above, we can use it to find the value of the nth term, as well as find the term number that has a specific value.

Let's first find the value of an nth term.

EXAMPLE For the sequence {7, 11, 15, 19, 23, 27, 31, ... }, find the value of the 18th term using the formula $a_n = 7 + 4(n-1)$.

Recall that the variable *n* stands for the term number. So to find the value of the 18th term, we substitute 18 in for *n* and solve:

$a_n = 7 + 4(n-1)$	Substitute 18 in for *n*

$a_{18} = 7 + 4(18 - 1)$ Subtract 1 from 18

$a_{18} = 7 + 4(17)$ Multiply 4 times 17

$a_{18} = 7 + 68$ Add 7 and 68

$a_{18} = 75$ Our solution

The 18th term in the sequence {7, 11, 15, 19, 23, 27, 31, ... } is 75.

Now let's find the term number that has a specific value.

EXAMPLE For the sequence {7, 11, 15, 19, 23, 27, 31, ... }, use the formula $a_n = 7 + 4(n - 1)$ to find the term that has the value of 255.

Here, we need to solve for *n*, given that a_n is 255.

$a_n = 7 + 4(n - 1)$ Substitute 255 in for a_n

$255 = 7 + 4(n - 1)$ Distribute 4 into $(n - 1)$

$255 = 7 + 4n - 4$ Combine 7 and -4

$255 = 4n + 3$ Subtract 3 from both sides

$252 = 4n$ Divide both sides by 4

$63 = n$ Our solution

255 is the 63rd term in the sequence {7, 11, 15, 19, 23, 27, 31, ... }

6. Real-World Arithmetic Sequence

Suppose you opened a deposit account. In the first month, you made an initial deposit of $1800. You plan to contribute $150 a month after your initial deposit. The account does not pay any interest.

After how many months will you have a total of $3000?

The amount in your deposit account can be modeled using the formula for an arithmetic sequence, $a_n = a_1 + d(n - 1)$. In this formula:

- a_n is the value after n months, in this case, $3000
- a_1 is the initial balance, in this case, $1800
- d is the common difference (or steady monthly deposits), in this case, $150

We want to find n, the number of months. Let's plug in the above values to solve.

$a_n = a_1 + d(n-1)$	Use the formula for arithmetic sequences and substitute the values: $a_n = 3000$, $a_1 = 1800$, and $d = 150$
$3000 = 1800 + 150(n-1)$	Distribute 150 into $(n-1)$
$3000 = 1800 + 150n - 150$	Combine 18000 and -150
$3000 = 1650 + 150n$	Subtract 1650 from both sides
$1350 = 150n$	Divide both sides by 150
$9 = n$	Our solution

It will take 9 months to have a total of $3,000 in your deposit account. You can use a table to validate the solution.

Month	Deposit Account Total
1	$1800
2	$1950
3	$2100
4	$2250
5	$2400
6	$2550
7	$2700
8	$2850
9	$3000

SUMMARY

In math, a **sequence** is a set of numbers in a particular order. We defined an **arithmetic sequence** as a set of numbers in numerical order with a common difference. The common difference is the numerical distance between any two consecutive terms in an arithmetic sequence. We can use the **formula for finding a term in an arithmetic sequence** to find larger terms in the sequence, such as the 50th or 400th term. We can use this formula when **writing a formula for an arithmetic sequence**. We can also **use the formula find a_n or n**.

TERMS TO KNOW

Arithmetic Sequence

A set of numbers in numerical order, with a common difference between each term.

Common Difference

The numerical distance between any two consecutive terms in an arithmetic sequence; a constant value.

Term (in a Sequence)

Refers to the place or order of a number in a sequence, such as first, second, third, etc.

FORMULAS TO KNOW

Arithmetic Sequence

$a_n = a_1 + d(n - 1)$