Introduction to Exponential Equations

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WHAT'S COVERED

In this lesson, you will learn how to evaluate an exponential function for a given value of *x*. Specifically, this lesson will cover:

1. Polynomial Equations versus Exponential Equations
2. Restrictions to the Base of Exponential Equations
3. Exponential Relationships in Tables

1. Polynomial Equations versus Exponential Equations

You may be familiar with equations that contain exponents. Take the polynomial equation $y = 2x^2 + 5x - 7$, for example. We see variables, and we see exponents. However, this is not an exponential equation. An **exponential equation** contains a variable exponent. This means that our variable *x*, for example, is part of the expression for the exponent, not a base number.

Generally, we say that an exponential equation is given by: $y = ab^x$

FORMULA TO KNOW

Exponential Equation

$y = ab^x$

We have a base number, *b*, being raised to a variable exponent, *x*. We also have an a-value in front, which is a scalar multiplier to the exponential expression.

TERM TO KNOW

Exponential Equation

An equation involving a constant value raised to a variable power.

2. Restrictions to the Base of Exponential Equations

The domain (or values that *x* is allowed to take on) is all real numbers, from negative infinity to positive infinity. As such, there are some restrictions to the base. These are values for *b* that come into direct conflict with the all-real number domain:

- The base must NOT be negative. This is because certain values of *x* would be excluded from the domain. We can think of raising -5 to the power of 1/2, or $-5^{\frac{1}{2}}$. This results in a non-real number (we can equivalently think of this as the square root of negative 5, or $\sqrt{-5}$).
- The base must NOT be zero. This is because it excludes all negative values from *x*. Take for example, 0 raised to the power of -2, or 0^{-2}. This can be thought of as 1 divided by 0^2, or $\frac{1}{0^2}$, which is division by zero and would result in a non-solution.
- The base CANNOT equal 1. This doesn't represent a restriction to the domain, but if the base were 1, we actually wouldn't have an exponential relationship at all. Rather, the relationship would be linear.

BIG IDEA

In an exponential equation $y = ab^x$, the base b has to be greater than zero and cannot equal 1 ($b > 0$ and $b \neq 1$).

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3. Exponential Relationships in Tables

Let's examine the pattern for exponential equations by looking at the relationship in a table.

EXAMPLE Find the following values for the exponential equation $y = 3 \cdot 2^x$.

When filling out values in a table, we apply the exponent first, and then multiply by the outside factor. Let's first look at the positive values of *x*:

x	$y = 3 \cdot 2^x$	*y*
1	$y = 3 \cdot 2^1 = 3 \cdot 2 = 6$	6
2	$y = 3 \cdot 2^2 = 3 \cdot 4 = 12$	12
3	$y = 3 \cdot 2^3 = 3 \cdot 8 = 24$	24

Now let's look at negative values of x:

x	$y = 3 \cdot 2^x$	y
-1	$y = 3 \cdot 2^{-1} = 3 \cdot \frac{1}{2^1} = 3 \cdot \frac{1}{2} = \frac{3}{2}$	$\frac{3}{2}$
-2	$y = 3 \cdot 2^{-2} = 3 \cdot \frac{1}{2^2} = 3 \cdot \frac{1}{4} = \frac{3}{4}$	$\frac{3}{4}$
-3	$y = 3 \cdot 2^{-3} = 3 \cdot \frac{1}{2^3} = 3 \cdot \frac{1}{8} = \frac{3}{8}$	$\frac{3}{8}$

Notice how the y-values from 1 to 3 increase at an increasing rate, and the y-values from -1 to -3 decrease at a decreasing rate, rather than a constant rate like in a linear equation.

BIG IDEA

In exponential equations, the exponent is applied to the base number first, and then multiplied by the scalar value in front of the exponential expression.

SUMMARY

The difference between **polynomial equations versus exponential equations** is that polynomial equations have variables as the base, such as x^2, and exponential equation have variables as the exponent, such as 5^x is an equation involving a constant value raised to a variable power. The general form of an exponential equation is a times b to the x power.

The domain of an exponential function is all x values. The range is all y values greater than 0 for a greater than 0, and all y values less than 0 for a less than 0. However, there are **restrictions to the base of exponential equations**, such that the base must be greater than 0 and cannot equal 1. Looking at **exponential relationships in a table**, you can see that exponential functions, as compared to linear functions, have graphs that increase at an increasing rate or decrease at a decreasing rate.

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TERMS TO KNOW

Exponential Equation

An equation involving a constant value raised to a variable power.

FORMULAS TO KNOW

Exponential Equation

$y = ab^x$