Introduction to Functions

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WHAT'S COVERED

In this lesson, you will learn how to identify elements of functions. Specifically, this lesson will cover:

1. Relations

When we discuss the term relation in mathematics we are referring to mapping how one set of values, called the input or domain, relates to another set of values, called the output or range. There are many examples of **relations** that you may come across in everyday encounters.

EXAMPLE One instance may be a list of companies and the number of employees each company has working for them. The input would be companies and the output would be the number of employees.

Oftentimes though in math, we try to map one set of numbers to another set of numbers and try to determine if there is a relationship between these numbers so that we can make predictions about future events.

EXAMPLE Suppose we wanted to determine a relationship between age and height. Below is a sample of the type of data that we might find.

x: Age (years)	5	10	15	15	20	25	25	30	35
y: Height (inches)	24	48	70	72	73	74	72	71	70

Notice here how we are relating age to height: a 5-year-old has a height of 24 inches, a 10-year-old has a height of 48 inches, and so on. We say that the age represented by x represents elements in the **domain** and y represents elements in the **range**.

Notice in the table above, we have two values in the domain that plot to the same value in the range. Both inputs 15 and 35 have an output of 70. We also have the same values of the domain, 15, that plot to different values in the range, 70 and 72. This is an important observation to make when distinguishing a relation from a function. Let's take a look at what we mean by the term "function" in mathematics in the next section.

TERMS TO KNOW

Domain
The set of all input values of a function or relation.

Range

The set of all output values of a function or relation.

Relation

A relationship between two sets of values, such that each element in one set corresponds to an element in the other set.

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2. Functions

A **function** is a special type of relation where every element, x, in the domain corresponds to exactly one element in the range. Unlike the previous example where we had the age 15 map to two different heights, 70 and 72, in a function, we cannot have this happen.

↪ EXAMPLE Consider this function where the volume of water in a container and the container's weight are shown in the table below.

x: Volume of Water (liters)	0.25	0.50	0.75	1
y: Weight of Water (grams)	250	500	750	1000

Notice in the above example that each value in the domain, x, maps to only one value in the range, y. However, you can have an element, y, in the range that corresponds to more than one element in the domain.

HINT

We can say that a function is a relation, but a relation is not necessarily always a function.

Determine if each relation in the table below is a function.

<table>
<tr><th>Relation</th><th>Function?</th><th>Explanation</th></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>-12</td><td>1</td></tr><tr><td>-10</td><td>9</td></tr><tr><td>0</td><td>-3</td></tr><tr><td>5</td><td>-7</td></tr></table></td><td>Yes</td><td>Each x-value in the domain appears only once, which means each x-value has only one y-value in the range.</td></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>4</td><td>15</td></tr><tr><td>3</td><td>12</td></tr><tr><td>5</td><td>18</td></tr><tr><td>4</td><td>10</td></tr></table></td><td>No</td><td>There are two x-values in the domain that are the same, but have different y-values in the range. This is NOT a function because each x-value needs to have only one y-value.</td></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>10</td></tr><tr><td>1</td><td>15</td></tr><tr><td></td><td></td></tr></table></td><td>Yes</td><td>Even though there are two x-values that have the same y-value, each x-value has only one y-value in the range.</td></tr>
</table>

2	15
3	20

TERM TO KNOW

Function

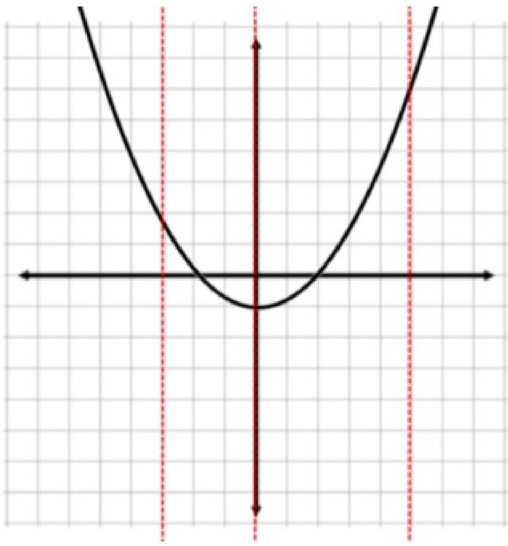
A special type of relation, in which every element in the domain corresponds to exactly one element in the range.

3. Vertical Line Test

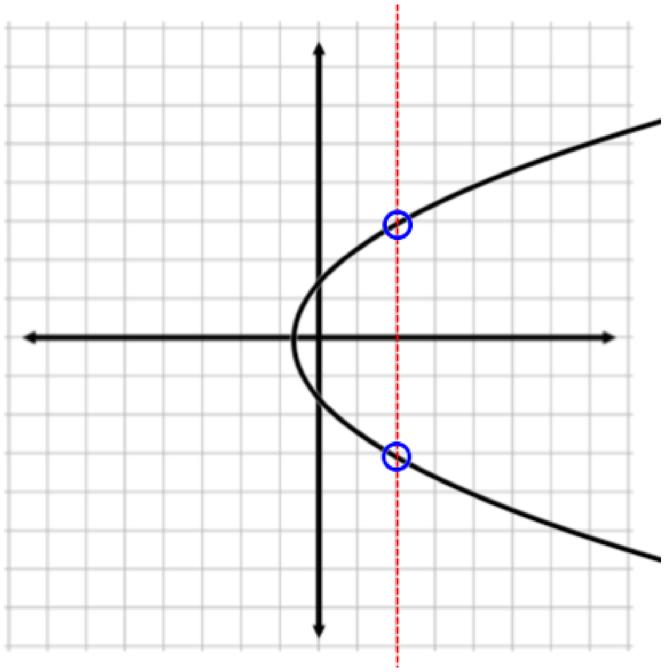
There are other ways to determine if a given set of data represents a function. One of the most common ways we do this is to graph the data on a coordinate plane and perform a vertical line test. To perform a vertical line test, we take a vertical line (a line that goes up and down) and move it across the entire graph. If the vertical line touches the function at only one point across the entire graph, then it passes the vertical line test.

If a function passes the vertical line test, then we can claim that the graph represents a function.

EXAMPLE Consider the graph below. Notice that in this graph shown above, several vertical lines were drawn in and no line touches the graph more than once. Therefore, we can conclude that this represents a function.



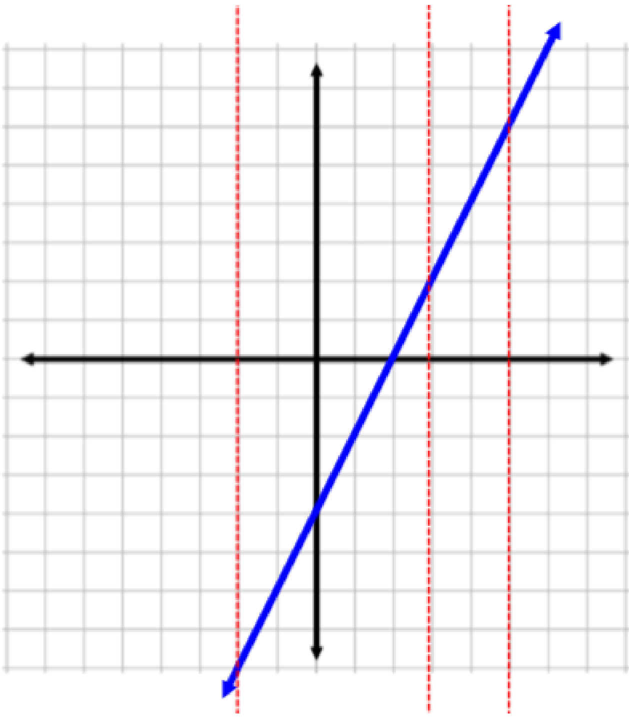
EXAMPLE On the other hand, suppose we have a horizontal parabola, as shown below. If we perform the vertical line test on the graph of this relation, is it also a function?



Notice that there is more than one point at which a vertical line hits the curve. Therefore, we can conclude that this graph does NOT represent a function.

4. Function Notation

Let's now take a look at the notation we use to represent functions. Suppose we have the linear relation, $y = 2x - 4$. Notice that, if we were to graph this line and apply the vertical line test to it, we can confirm that this is a function.



When working with functions we like to write those functions in a notation that lets us know that we are working with a function, and what inputs the outputs of the function are dependent on. In this process we rewrite $y = 2x - 4$ as $f(x) = 2x - 4$. Note that here y and $f(x)$ are the same thing. We read this statement as, "f of *x* is equal to 2x minus 4”, which means that this function, generically represented by *f*, is dependent on the input *x*.

HINT

Be careful that you do not mistake $f(x)$ for *f* multiplied by *x*. When we write $f(x)$, we are noting that we are working with functions, not that we are multiplying quantities. When dealing with functions, $f(x)$ always means that the output of the function is dependent on the input *x*.

When solving this above function for a given value of *x*, instead of saying finding the value of y when $x = a$, as we have done before, we say "find $f(a)$". This is read as "*f* of a". While the notation is slightly different, the process we use is generally the same. Let’s look at an example:

EXAMPLE Solve the equation $y = 2x - 4$ and function $f(x) = 2x - 4$ when $x = 3$.

$y = 2x - 4$	$f(x) = 2x - 4$	Problem Statement
$y = 2(3) - 4$	$f(3) = 2(3) - 4$	Substitute in $x = 3$
$y = 6 - 4$	$f(3) = 6 - 4$	Simplify
$y = 2$	$f(3) = 2$	Our solution

HINT

When asked to solve a function $f(x)$ for the quantity "*a*", simply substitute all *x* variables in the function with "*a*" and simplify.

SUMMARY

A **relation** is a relationship between two sets that matches elements of one set with elements of another set. The first value in the ordered pair, *x*, represents the elements in the domain, and the second value, *y*, represents elements in the range. A **function** is defined as a relation in which every element in the domain corresponds to exactly one element in the range. If a graph represents a function, at any value for *x*, the curve of the function touches or crosses any vertical line only once, which is called the **vertical line test**. **Function notation** is used to evaluate a function for a given value in the domain of the function.

TERMS TO KNOW

Domain
The set of all input values of a function or relation.

Function
A special type of relation, in which every element in the domain corresponds to exactly one element in

the range.

Range

The set of all output values of a function or relation.

Relation

A relationship between two sets of values, such that each element in one set corresponds to an element in the other set.