Introduction to Geometric Sequences

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WHAT'S COVERED

In this lesson, you will learn how to determine the formula for a given geometric sequence. Specifically, this lesson will cover:

1. Geometric Sequence Defined

A sequence is a set of numbers in numerical order. You may be familiar with arithmetic sequences, which have a common difference between each term, a constant number that is either added or subtracted as we go from one term to the next.

EXAMPLE Consider the arithmetic sequence $\{3, 5, 7, 9...\}$, with a common ratio of 2, where we add 2 to get the value of the next term: 3 plus 2 is 5, 5 plus 2 is 7, 7 plus 2 is 9, etc.

With **geometric sequences**, there is a **common ratio** between each term. Similar to arithmetic sequences, the common ratio is a constant value; however it is not added or subtracted from one term to the next, it is multiplied by the preceding term to continue the sequence.

EXAMPLE Consider the geometric sequence $\{3, 6, 12, 24, 48...\}$, with a common ratio of 2. We can multiply a term by 2 to get the value of the next term: 3 times 2 is 6, 6 times 2 is 12, 12 times 2 is 48, etc. The next term in the sequence would be 96, and 192 would follow that.

TERMS TO KNOW

Geometric Sequence
A set of numbers in numerical order, with a non-zero common ratio between each term.

Common Ratio

The ratio between any two consecutive terms in a geometric sequence, a constant value.

2. Divergent and Convergent Sequences

The previous sequence was also an example of a divergent sequence. A **divergent sequence** is a sequence whose terms do not have a finite limit; they tend toward positive or negative infinity.

Since we are continuously multiplying each term in the sequence by 2 in the above example, we know that the terms will continue to increase in value, making the limit of values positive infinity. Even if the common ratio were -2, the terms would alternate between positive and negative, but would still grow in absolute magnitude. Divergent sequences are characterized by a common ratio greater than 1 or less than -1.

EXAMPLE Consider if we have a geometric sequence with a common ratio of 5. The terms will tend toward positive infinity as we continue:

$\{5, 25, 125, 625\ldots\}$

In contrast, some geometric sequences are convergent. The terms in **convergent sequences** do have a limit. Convergent sequences are characterized by a common ratio between -1 and 1.

EXAMPLE Consider if we have a geometric sequence with a common ratio of 0.5, as in the example below:

$\{768, 384, 192, 96\ldots\}$

In this sequence, each term is multiplied by 0.5, or in other words, cut in half, as the sequence continues. Eventually, the term would be virtually zero, thus the sequence is convergent. Even if the common ratio were -0.5, the terms would still converge to zero, even though they would alternate between positive and negative.

BIG IDEA

Divergent and convergent sequences are characterized by their common ratio, *r*:

- If $|r| > 1$, the sequence is divergent.
- If $|r| < 1$, the sequence is convergent.

TERMS TO KNOW

Divergent Sequence

A sequence whose terms do not have a finite limit; they tend toward positive or negative infinity.

Convergent Sequence

A sequences whose terms have a finite limit; they tend toward a specific value.

3. Formula for Geometric Sequences

Let's return to the geometric sequence: $\{3, 6, 12, 24, 48...\}$. It was easy enough to find the value of the next two terms, by multiplying 48 by 2 to get 96, and then by 2 again to get 192. What if we wanted to find the value of the 78th term? Or even the 115th therm? Certainly finding the value of that term by continuously multiplying by 2 is inefficient.

To find the nth term of a sequence, we can use the **geometric sequence formula**:

FORMULA TO KNOW

Geometric Sequence

$a_n = a_1 \cdot r^{n-1}$

In this formula:

- a_n is the value of the nth term.
- a_1 is the value of the first term.
- r is the common ratio.
- n is the term number.

4. Writing a Formula for a Geometric Sequence

EXAMPLE Consider the geometric sequence $\{8748, 2916, 972, 324...\}$.

How can we develop a formula to represent this sequence, so that we may use it to find the value of any term? We will need to identity a_1 and r:

- a_1: The value of the first term is 8748, so this will be a_1 in our formula.
- r: We need to calculate the common ratio, *r*. To do so, we will take any two consecutive terms, and divide the second by the first, for instance, $324 \div 972 = \frac{1}{3}$. This tells us that each term is multiplied by $\frac{1}{3}$ to get the value of the next term.

Plugging in 8748 for a_1 and $\frac{1}{3}$ for r, the formula for this sequence is:

$$a_n = 8748 \cdot \left(\frac{1}{3}\right)^{n-1}$$

5. Using the Formula to find a Term

EXAMPLE Use the above formula $a_n = 8748 \cdot \left(\frac{1}{3}\right)^{n-1}$ to find the value of the 7th term in the sequence.

Since we want to find the 7th term, we use plug 7 for *n* in our formula:

$a_n = 8748 \cdot \left(\frac{1}{3}\right)^{n-1}$	Plug in 7 for *n*
$a_n = 8748 \cdot \left(\frac{1}{3}\right)^{7-1}$	Evaluate the exponent
$a_n = 8748 \cdot \left(\frac{1}{3}\right)^{6}$	Raise $\left(\frac{1}{3}\right)$ to the 6th power
$a_n = 8748 \cdot \left(\frac{1^6}{3^6}\right)$	Evaluate fraction
$a_n = 8748 \cdot \left(\frac{1}{729}\right)$	Multiply 8748 and $\frac{1}{729}$
$a_n = \frac{8748}{729}$	Divide 8749 by 729
$a_n = 12$	Our solution

SUMMARY

A **geometric sequence** is a set of numbers in numerical order with a nonzero common ratio between each term. There are two types of sequences: **divergent and convergent sequences**. Divergent sequences have a common ratio, *r*, such that the absolute value of *r* is greater than 1. Convergent sequences have a common ratio such that the absolute value of *r* is less than 1. The **formula for geometric sequences** is a sub n equals a sub 1 times *r* to the *n* minus one power. When **writing a formula for a geometric sequence**, you need to determine the common ratio by dividing any term in the sequence by the term before it. You can also **use the formula to find any term** in the sequence.

TERMS TO KNOW

Common Ratio

The ratio between any two consecutive terms in a geometric sequence; a constant value.

Convergent Sequence

A sequence whose terms have a finite limit; they tend toward a specific value.

Divergent Sequence

A sequence whose terms do not have a finite limit; they tend toward ± infinity.

Geometric Sequence

A set of numbers in numerical order, with a non-zero common ratio between each term.

FORMULAS TO KNOW

Geometric Sequence

$a_n = a_1 \cdot r^{(n-1)}$