Introduction to Inequalities

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to identify the correct compound inequality for a given number line. Specifically, this lesson will cover:

1. What is an Inequality?

So far in this course, we have only talked about equalities; in other words, $2+3=5$ or $x=4$. In this lesson, we will cover inequalities. An **inequality** is a mathematical statement to show that one quantity is greater than or less than another quantity. The following outlines the different inequality symbols and their meanings:

Symbol	Translation	Example
$<$	Less than	$3<5$ 3 is less than 5
$>$	Greater than	$7>5$ 7 is greater than 5
$\leq$	Less than or equal to	$4\leq 5$ 4 is less than or equal to 5
$\geq$	Greater than or equal to	$8\geq 5$ 8 is greater than or equal to five

EXAMPLE If we have an expression such as $x<4$, this means our variable can be any number smaller than 4 such as -2, 0, 3, 3.9, or even 3.999999999, just as long as it is smaller than 4.

EXAMPLE If we have an expression such as $x\geq 2$, this means our variable can be any number greater than or equal to -2, such as 5, 0, -1, -1.9999, or even -2.

HINT

Can't remember which way the inequality symbol should be facing? Imagine it's a hungry animal's mouth and it always wants to eat the bigger value. The wide-open side of the inequality points to the larger value.

TRY IT

Identify the correct inequality symbol in the following examples:

4 _ 7 +

4 < 7

4 is smaller/less than 7. Have the inequality "mouth" face toward the larger value: 7.

-5 _ -12 +

-5 > -12

Negative 5 (-5) is greater than negative 12 (-12) because it is further to the right on the number line. Have the inequality “mouth” face toward the larger value: -5.

6 _ 8 +

6 < 8

6 is smaller/less than 8. Have the inequality “mouth” face toward the larger value: 8.

-4 _ -3 +

-4 < -3

-4 is smaller/less than negative 3 (-3). Have the inequality “mouth” face toward the larger value: - 3.

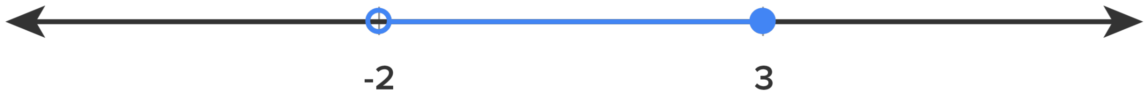
TERM TO KNOW

Inequality
A mathematical statement that two quantities are not equal in value.

2. Plotting Inequalities on a Number Line

Because we don’t have one set value for our variable, it is often useful to draw a picture of the solutions to the inequality on a number line. On a number line, we define specific intervals on the number line using symbols. An endpoint is where a range of values starts or ends.

EXAMPLE Let’s look at the interval on the number line below. Here is our number line with a highlighted range of values:



If we examine the highlighted portion of the number line, we can see that the highlighted portion starts at negative 2 and ends at positive 3. These are the endpoints of this interval: -2 and +3. Note there is an open circle at -2. This means that the interval includes the numbers between -2 and 3, but the interval does not include -2. So, -1, -1.7, and -1.99 are within the interval, but -2 is not within the interval.

We can also see that there is a filled-in circle or a closed circle at the positive 3 on the number line. This means that the interval ends at 3 and the endpoint 3 is included in the interval. So, 1, 2.95, and 3 are all within the interval.

In general, we will use open and close circles to indicate endpoints:

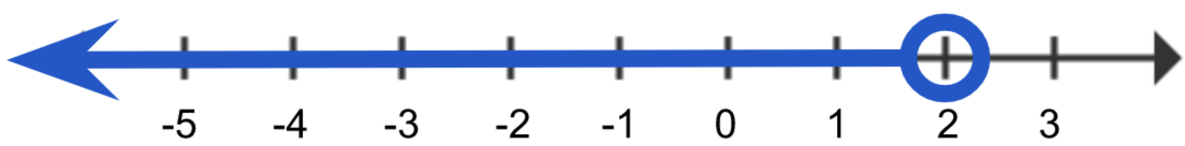
Open and Closed Circles for Intervals on a Number Line

Endpoint is Included in Interval	Endpoint is Not Included in Interval
●	○

2a. "Less Than" Inequalities

↪ EXAMPLE Graph the inequality $x < 2$.

For the "less than" inequality, $x < 2$, we know that x cannot be exactly equal to 2, but can be anything smaller. First, we show that x cannot be exactly equal to 2 by putting an open circle on the number line right at 2. An open circle means that the value for x cannot be exactly equal to the value that it's on top of. We also want to show that x can be any value less than 2. We can use an arrow pointing to the left, or pointing to the numbers that are less than 2. The region in the image represents all the solutions to the inequality, x is less than 2, and any value in the highlighted region is going to satisfy this inequality.



Keep in mind that if this inequality was $x \leq 2$, then we would have used a closed circle.

2b. "Greater Than" Inequalities

↱ EXAMPLE Graph the inequality $y \geq -1$.

For the "greater than or equal to" inequality, $y \geq -1$, we need to show that y can be exactly equal to -1. We can do that by putting a closed circle on the number line right at -1. Next, we want to show that y can also be anything greater than -1. We can use an arrow pointing to the right, towards the numbers that are greater than -1. This represents all the solutions to the inequality, $y \geq -1$. Any value of y in this highlighted region, including -1, is going to satisfy the inequality.

2c. Compound Inequalities

↱ EXAMPLE Graph the inequality $-3 < x \leq 2$.

For the compound inequality, $-3 < x \leq 2$, the values for x that are going to satisfy this inequality have to be bigger than -3 and less than or equal to 2. We start by putting an open circle at -3 because we see that x cannot be exactly equal to -3. Then we are going to use a closed circle at 2 because we know that x can be equal exactly to 2.

We want to show that x is greater than -3 but that x is also smaller than or equal to 2. We can see that our solution is going to be in between these two points. Using this highlighted region, we can see that any value that's in this region is going to satisfy the inequality. Any number greater than -3 or less than or equal to 2 will satisfy this inequality.

SUMMARY

When thinking about **what is an inequality**, we use inequalities to show that two quantities are not equal in value. We can use a number line to show a range of values that can satisfy an inequality expression. When we're **plotting an inequality on a number line**, we use an open circle or parentheses for the symbols "less-than" and "greater-than" and use a filled in circle or brackets for the symbols "less-than or equal to" and "greater-than or equal to."

TERMS TO KNOW

Inequality
A mathematical statement that two quantities are not equal in value.