Sophia

Introduction to Logarithms

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to solve a logarithmic equation. Specifically, this lesson will cover:

1. Relating Logarithms to Exponential Equations

There is an inverse relationship between **logarithms** and exponents. If we have the expression $3^x = 9$, we can gather that *x* equals 2, because 3 squared equals 9 ($3^2 = 9$).

As a logarithmic expression, we can write this equivalently as $\log_3(9) = 2$. This reads, "the log, base 3, of 9 is 2." The expression tells us that the base number, 3, must be raised to the power of 2 in order to equal 9.

In general, we can write the relationship between logarithms and exponents as follows:

$y = b^x$	Exponential equation
$\log_b(y) = x$	Logarithmic equation

Notice that *x* and *y* switched as being isolated onto one side of the equals sign. This is characteristic of inverse relationships. Also, note that the base to the exponential is the base of the logarithm.

EXAMPLE Rewrite the exponential equation $8 = 2^x$ as a logarithmic equation

$8 = 2^x$	Exponential equation
$\log_2(8) = x$	Logarithmic equation

HINT

If you know your powers of 2, you may be able to gather that $x = 3$ in this case.

TERM TO KNOW

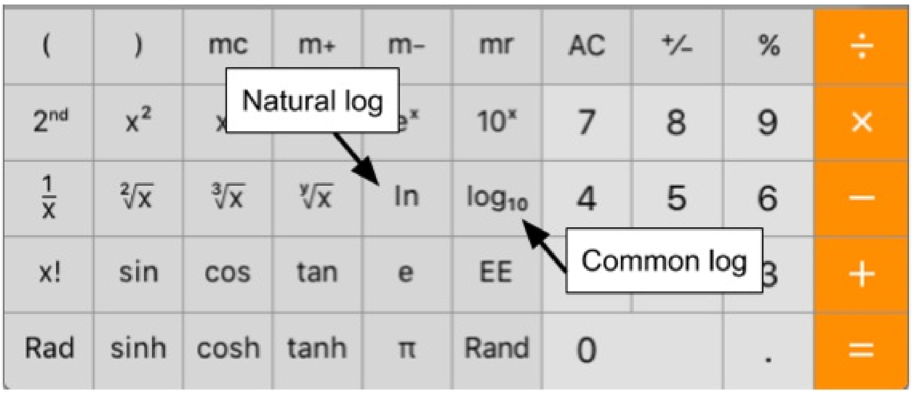
Logarithm

The inverse of a power, the logarithm describes how many times a number should be multiplied by itself to result in another number.

---

2. Common Log and Natural Log

If you have a scientific calculator that can compute logarithms, there are likely two kinds of log buttons on your calculator: one that simply says "log" and another that says "ln." The first button, "log" is known as the common log, while the other, "ln," is referred to as the natural log.



They are both logarithms, but their difference is in their base. Common log operates under a base of 10. So if you ever see expressions such as log(42) or log(67), the base of the log is 10.

HINT

Whenever a base is not explicitly written next to "log," it is assumed to be the common log, which is base 10.

The abbreviation "ln" comes from the Latin *logarithmus naturali*. The base of this logarithm is the mathematical constant "e". The constant "e", or Euler's constant, is approximately equal to 2.718282. If you have the natural log button (ln) on your calculator, definitely use it for the most accurate calculations. Otherwise, use the approximation 2.718282.

HINT

ln, or natural log, operates in base *e*, which is approximately equal to 2.718282. ln(x) and log$e$(x) are the same expressions.

---

3. Evaluating Logarithmic Expressions

We can use the relationship between exponential equations and logarithmic equations to evaluate expressions by thinking about how many times we must multiply a given number by itself to result in another given number.

↪ EXAMPLE Evaluate $\log_4(64)$.

$\log_4(64)$	Rewrite using exponents
$4^x = 64$	4 cubed results in 64
$4^3 = 64$	Write the solution to expression
$\log_4(64) = 3$	Our solution

→ EXAMPLE Evaluate $\log_3(243)$.

$\log_3(243)$	Rewrite using exponents
$3^x = 243$	3 raised to the 5th power is 243
$3^5 = 243$	Write the solution to expression
$\log_3(243) = 5$	Our solution

Notice how the bases are the same in both exponential and logarithmic form.

$\log_b(y) = x$	$y = b^x$
$\log_4(64) = 3$	$64 = 4^3$
$\log_3(243) = 5$	$243 = 3^5$

SUMMARY

In an exponential equation, a base number is raised to a variable power and represented as $y = b^x$. **Relating logarithms to exponents**, the input of the logarithmic function is the output of the exponential function, and the output of the logarithmic function is the input of the exponential function.

With logarithms, there are two types of log: **common log and natural log**. The common log is a logarithm with a base of 10. An expression that just has log and now base, like log(42), implies a base of 10. The natural log is a logarithm with a base *e*, where *e* is equal approximately to 2.718281. When evaluating logarithmic expression, you can rewrite using an exponential expression.

TERMS TO KNOW

Logarithm

The inverse of a power, the logarithm describes how many times a number should be multiplied to result in another number.