Introduction to Quadratic Equations

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to identify a parabola given a vertex and solution set. Specifically, this lesson will cover:

1. Quadratics as Second Degree Polynomials

A **quadratic** is a second-degree polynomial, which means that in the expression of a quadratic, there will be no more than 2 x-terms being multiplied together. In expanded form, the highest exponent you will see is 2, and in factored form, there will only be two factors of *x*.

EXAMPLE The expression $(x+3)(x-2)$ is a quadratic, but the expression $(x+3)(x-2)(x+1)$ is not because there are 3 factors of *x*.

TERM TO KNOW

Quadratic

A second-degree polynomial, with an x-squared term as its highest degree term.

2. Forms of Quadratic Equations

There are several different ways to write a quadratic equation. We are going to cover standard form, vertex form, and factored form:

2a. Standard Form

Standard form is the expanded form of quadratic expressions. There is an x-squared term, an x-term, and a constant term. We use the coefficients a, b, and c, which are the same coefficients used to solve quadratic equations using the quadratic formula.

FORMULA TO KNOW

Standard Form

$$y = ax^2 + bx + c$$

2b. Vertex Form

Vertex form is ideal for graphing quadratic equations because it provides information about the parabola's vertex readily in its equation. The variables h and k represent the x- and y-coordinates to the vertex. The vertex is the point (h, k).

FORMULA TO KNOW

Vertex Form

$$y = a(x - h)^2 + k$$

2c. Factored Form

Equations in factored form allow us to easily identify the x-intercepts of the parabola, which we will later discuss as solutions to the quadratic equation. x_1 and x_2 represent x-values at which y is equal to zero.

FORMULA TO KNOW

Factored Form

$$y = a(x - x_1)(x - x_2)$$

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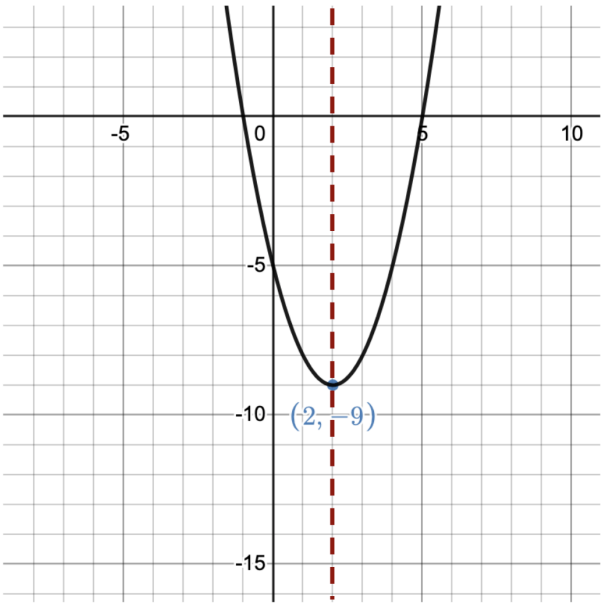
3. Parabolas

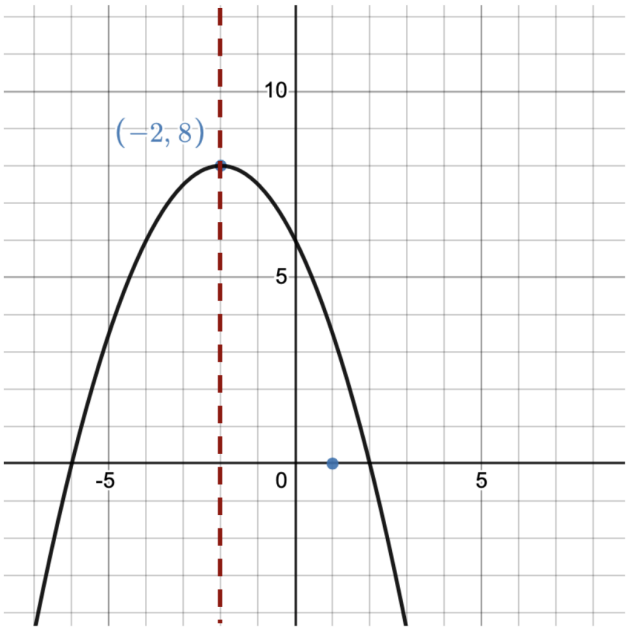
When quadratic equations are graphed, we call the curve a **parabola**. It has a distinct U shape to it (or an upside-down U shape if the parabola opens downward).

There is also either a minimum or a maximum point (also dependent upon which direction the parabola opens). This maximum or minimum point is known as the **vertex** of the parabola, and it lies on a line of symmetry to the graph. This means that one half of the parabola can be reflected about that line of symmetry to match up perfectly with the other half of the parabola.

EXAMPLE Here are a couple of graphs of parabolas. See if you can spot the vertex, and notice its symmetry:



U-Shape
Opens upward
Vertex: (2,-9)



Upside-down U shape
Opens downward
Vertex: (-2,8)

HINT

The leading coefficient to the equation (the value of "a" in each of the three forms listed in the section above) determines whether the parabola opens up or down. If a is positive, the parabola opens up (and has a U shape). If a is negative, the parabola opens down (and has an upside-down U shape). You'll learn more about this in a later lesson.

TERMS TO KNOW

Parabola

The shape of a quadratic equation on a graph; it is symmetric at the vertex.

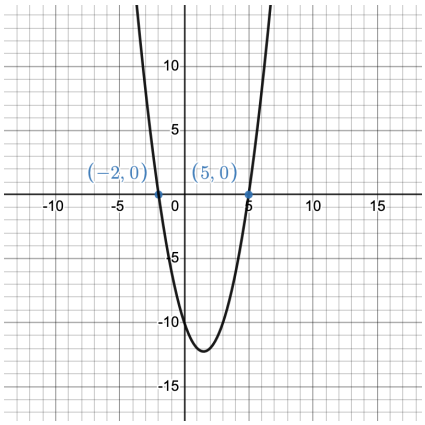
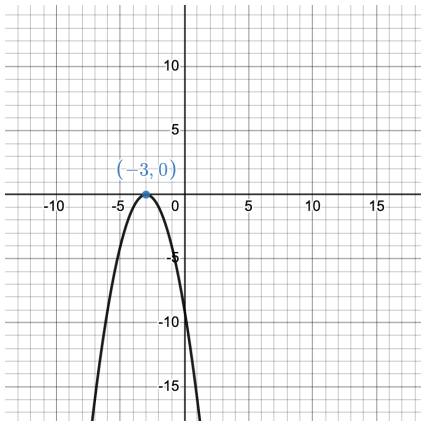
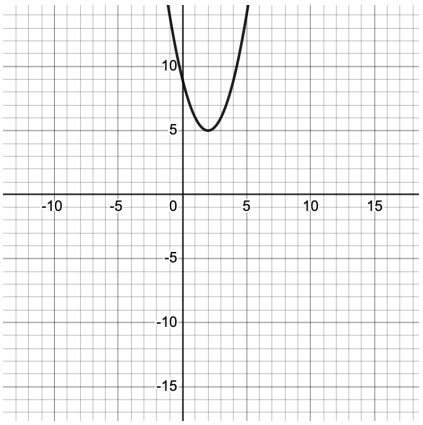
Vertex (of a Parabola)

The maximum or minimum point of a parabola located on the axis of symmetry.

4. Solutions to Quadratic Equations

A solution to a quadratic equation is also referred to as a zero, or a root. This is because solutions are x-values that make *y* equal to zero. Graphically, these are x-intercepts to the parabola. Quadratic equations can have zero, one, or two real solutions. There will never be three real solutions to the equation. This is because parabolas can intersect the x-axis at most 2 times.

EXAMPLE Take a look that the following graphs of parabolas and notice their solutions, or the spot where they intersect the x-axis.

Two Solutions	One Solution	No Solutions
		
Solutions: (-2,0) and (5,0)	Solutions: (-3,0)	No Real Solutions

There are several different methods for solving a quadratic equation. The most common ways are by using the Zero Factor Property and using the Quadratic Formula. These methods are covered in greater detail elsewhere, but in general:

- The Zero Factor Property takes advantage of the fact that anything multiplied by zero equals zero. We set each factor of the quadratic equal to zero and solve for *x*. This is the ideal method when working with equations written in factored form.
- The Quadratic Formula uses the coefficients a, *b*, and *c* from equations written in standard form, and set equal to zero. Plugging in these values, and performing the algebraic steps to solve for *x* will give solutions to the quadratic that might not be easily solved by factoring.

FORMULA TO KNOW

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SUMMARY

When we think of quadratics, we can think of **quadratics as second degree polynomials**. There are three different **forms of quadratic equations**: standard, vertex, and factored. The graph of a quadratic equation is called a **parabola**. Parabolas have either a minimum or a maximum point, which is called the vertex. The **solutions to quadratic equations** can also be called a root or a 0. The solution represents the x-intercepts of the parabola on a graph. And a quadratic equation can be solved algebraically by substituting 0 for *y* in the equation, and solving for *x*.

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TERMS TO KNOW

Parabola

The shape of a quadratic equation on a graph; it is symmetric at the vertex.

Quadratic

A second-degree polynomial, with an x-squared term as its highest degree term.

Vertex (of a Parabola)

The maximum or minimum point of a parabola located on the axis of symmetry.

FORMULAS TO KNOW

Factored Form of a Quadratic Equation

$$y = a(x - x_1)(x - x_2)$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Standard Form of a Quadratic Equation

$$y = ax^2 + bx + c$$

Vertex Form of a Quadratic Equation

$$y = a(x - h)^2 + k$$