Introduction to Rational Expressions

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WHAT'S COVERED

In this lesson, you will learn how to identify the domain restrictions of a rational expression. Specifically, this lesson will cover:

1. What is a Rational Expression?

A **rational expression** is a ratio of two algebraic expressions. Specifically, we have a polynomial expression in the numerator and a polynomial expression in the denominator of a fraction. As functions, we can write a rational function as:

$$f(x) = \frac{p(x)}{q(x)}$$

So we can refer to $p(x)$ as the polynomial in the numerator and $q(x)$ as the polynomial in the denominator.

EXAMPLE $\frac{3x^2-6x+4}{x+5}$

EXAMPLE $\frac{(x-2)(x+3)}{(x+2)(x-1)}$

EXAMPLE $\frac{5}{x+1}$

TERM TO KNOW

Rational Expression
A fraction in which the numerator and denominator are polynomials.

2. Domain of Rational Expressions

Because we have a denominator in rational expressions, we sometimes set restrictions on what the denominator can equal. In general, we cannot divide by zero, otherwise, the expression is undefined. Therefore, variables can take on any value, provided that it does not make the denominator equal to zero. This represents a domain restriction.

Below, we are going to practice identifying domain restrictions by looking at rational expressions in different forms. In each example, we'll talk about how we can go about identifying x-values that make the denominator equal to zero.

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3. Identifying Domain Restrictions

To find domain restrictions, we focus on the denominator of the expression.

↱ EXAMPLE Find the domain restrictions on the expression $\frac{(2x-2)(x+3)}{(x-1)(2x+4)}$.

The denominator is a polynomial expression in factored form. To find values of x which make the denominator equal to zero, we can set each factor equal to zero, and solve for x:

$\frac{(2x-2)(x+3)}{(x-1)(2x+4)}$	Set denominator equal to zero
$(x-1)(2x+4)=0$	Set each factor equal to zero
$x-1=0, \quad 2x+4=0$	Solve each equation
$x=1, \quad x=-2$	Our domain restrictions

We can conclude that x is not allowed to be -2 or 1.

↱ EXAMPLE Find the domain restrictions on a rational expression in which the polynomials are written in expanded form $\frac{2x^2+5x+3}{x^2-8x+15}$.

It would be ideal if we could factor the denominator so that we can solve using a similar method. Notice that it is not necessary to factor the numerator at all, because we really do not care what the value of the numerator is. All that matters is that we find values for x that make the denominator equal to zero.

We notice that the denominator is a quadratic expression, so we will try to factor it. If we cannot factor easily, using the quadratic formula is also an option. Fortunately, factoring the quadratic is possible, and it isn't terribly difficult. Since the constant term is positive, but the x-term coefficient is negative, we are looking for two negative integers that sum to -8, but multiply to 15. These happen to be -3 and -5:

$\frac{2x^2+5x+3}{x^2-8x+15}$	Set denominator equal to zero
$x^2-8x+15=0$	Factor the quadratic
$(x-3)(x-5)$	Set each factor equal to zero
$x-3=0, \quad x-5=0$	Solve each equation
$x=3, \quad x=5$	Our domain restrictions

We can conclude that x is not allowed to be 3 or 5.

↪ EXAMPLE Find the domain restriction on a rational expression with polynomials expressed in factored form $\frac{(2x-1)(x+3)}{(x+3)(x-2)}$.

Before we start, we notice common factors $(x+3)$ in both the numerator and denominator that can cancel out. We might think that because this factor can be canceled away, we can ignore it completely. This is not true. When canceling out factors in rational expressions, it is important to still consider them when finding domain restrictions. We must also set the canceled-out factors equal to zero and solve for x.

$\frac{(2x-1)(x+3)}{(x+3)(x-2)}$	Set denominator equal to zero
$(x+3)(x-2)=0$	Set each factor equal to zero
$x+3=0, \quad x-2=0$	Solve each equation
$x=-3, \quad x=2$	Our domain restriction

We can conclude that x is not allowed to be 2 or –3.

BIG IDEA

The denominator of rational expressions is not allowed to be zero. Therefore, we must set the denominator equal to zero, and solve for any x-values that make the denominator equal to zero. If no such solutions exist, there are no restrictions to what x can be.

SUMMARY

A **rational expression** is a ratio of two algebraic expressions, specifically with a polynomial expression in the numerator and denominator. When determining the **domain of rational expressions**, the expression in the denominator is not allowed to evaluate to zero or else the entire expression is undefined. Because of this, rational expressions have **domain restrictions** or values that the variables may not be.

TERMS TO KNOW

Rational Expression
A fraction in which the numerator and denominator are polynomials.