Introduction to Slope

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to identify the slope of a line perpendicular to a given line. Specifically, this lesson will cover:

1. The Slope of a Line

As we graph lines, we will want to be able to identify different properties of the lines we graph. One of the most important properties of a line is its slope. **Slope** is a measure of steepness. A line with a large slope, such as 25, is very steep. A line with a small slope, such as 0.10 is very flat. We will also use slope to describe the direction of the line. A line that goes up from left to right will have a positive slope and a line that goes down from left to right will have a negative slope.

As we measure steepness, we are interested in how fast the line rises compared to how far the line runs. For this reason we will describe slope as the fraction $\frac{\text{rise}}{\text{run}}$. Rise would be a vertical change, or a change in the y-values. Run would be a horizontal change, or a change in the x-values. So another way to describe slope would be the fraction $\frac{\text{change in } y}{\text{change in } x}$.

TERM TO KNOW

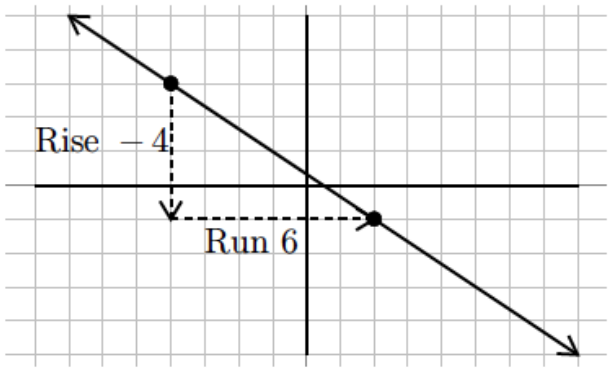
Slope

The steepness of a line; found by dividing the change in y-coordinates by the change in x-coordinates from any two points on a line.

1a. Using a Graph

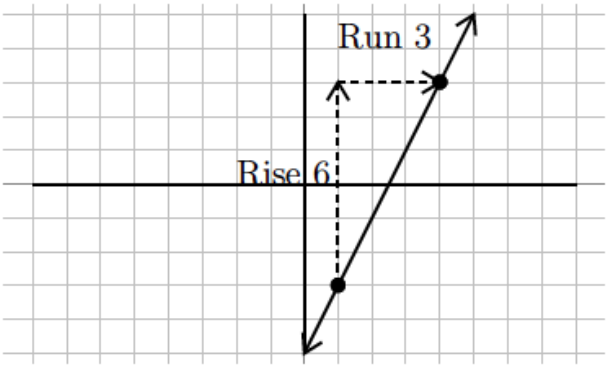
It turns out that if we have a graph, we can draw vertical and horizontal lines from one point to another to make what is called a slope triangle. The sides of the slope triangle give us our slope.

EXAMPLE Find the slope of the following line.



To find the slope of this line, we will consider the rise, or vertical change, and the run, or horizontal change. Drawing these lines makes a slope triangle that we can use to count from one point to the next. This graph goes down 4, right 6. This is a rise of -4 and a run or 6. As a fraction it would be, $\frac{-4}{6}$. Reduce the fraction to get $\frac{-2}{3}$, so the slope is $\frac{-2}{3}$.

EXAMPLE Find the slope of the following line.



To find the slope of this line, the rise is up 6, the run is to the right 3. Our slope is then written as a fraction, rise over run, or $\frac{6}{3}$. This fraction reduces to 2, so the slope is 2.

1b. Using Two Points

We can find the slope of a line through two points without seeing the points on a graph. We can do this using a slope formula. If the rise is the change in y-values, we can calculate this by subtracting the y-values of a point. Similarly, if the run is a change in the x-values, we can calculate this by subtracting the x-values of a point. So we can say the following:

The slope of a line through (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

When mathematicians began working with slope, it was called the modular slope. For this reason, we often represent the slope with the variable m. Now we have the following for slope.

FORMULA TO KNOW

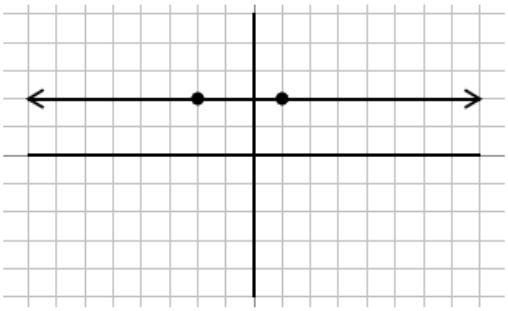
Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2. Slopes of Vertical and Horizontal Lines

There are two special lines that have unique slopes that we need to be aware of.

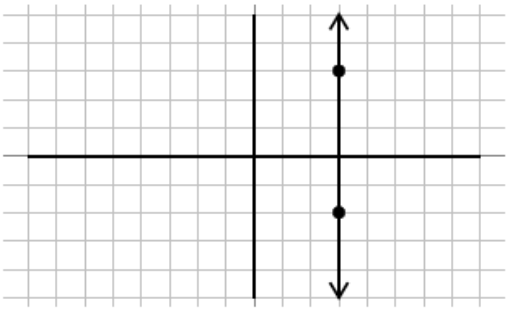
The line below is a horizontal line. For this line, the slope is not steep at all, in fact, it is flat. Therefore it has a zero slope.



BIG IDEA

All horizontal lines have a zero slope.

This next line is a vertical line. For this line, the slope can't get any steeper. It is so steep that there is no number large enough to express how steep it is so this is an undefined slope.



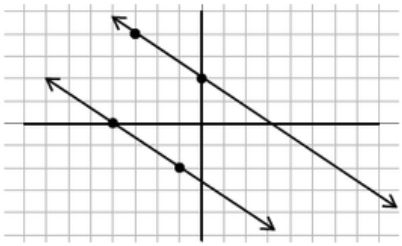
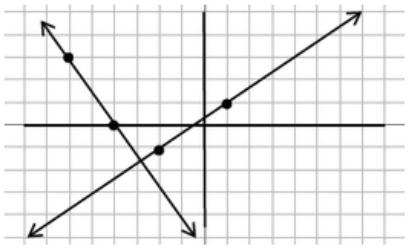
BIG IDEA

All vertical lines have an undefined slope.

As you can see there is a big difference between having a zero slope and having no slope or an undefined slope. Remember, slope is a measure of steepness.

3. Slopes of Parallel and Perpendicular Lines

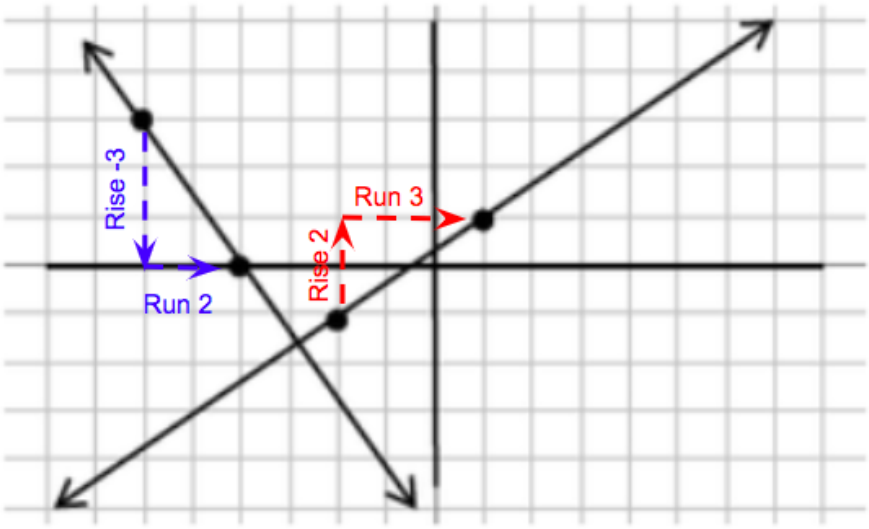
There is an interesting connection between the slope of lines that are **parallel** and the slope of lines that are **perpendicular** (meet at a right angle).

Type of Line	Description	Slope	Example
Parallel	Lines that never intersect	Identical slopes	
Perpendicular	Lines that intersect at a right angle	Opposite reciprocal slopes	

As the above graphs illustrate, parallel lines have the same slope, and perpendicular lines have opposite (one positive, one negative). In fact, perpendicular lines have reciprocal (flipped fraction) slopes. We can use these properties to make conclusions about parallel and perpendicular lines.

Let's take a further look at the example above with perpendicular lines.

EXAMPLE



One line has a rise of -3 and a run of 2, so the slope is $\frac{-3}{2}$. The other line that is perpendicular has a rise of 2 and a run of 3, so the slope of this line is $\frac{2}{3}$. These two slopes are opposite reciprocals. They are opposite meaning one slope is positive while the other is negative. The slopes are also reciprocals meaning their fractions are flipped.

BIG IDEA

When two lines are parallel, the slopes are the same. When two lines are perpendicular, the slopes are opposite reciprocals.

See how slopes of parallel and perpendicular lines compare in the following table:

Slope of Line 1	Slope of Line 2	Relationship	Explanation
$\frac{5}{12}$	$-\frac{12}{5}$	Perpendicular	The reciprocal of $\frac{5}{12}$ is $\frac{12}{5}$, and then make it negative!
3	3	Parallel	Since they both have the same value for the slope, they are parallel lines.
$\frac{3}{4}$	$\frac{4}{3}$	Neither	Although the fractions are reciprocals, they are not opposites.
$-\frac{1}{4}$	4	Perpendicular	The reciprocal of $-\frac{1}{4}$ is $-\frac{4}{1}$, or just 4. Then make it positive!
$\frac{3}{17}$	$-\frac{19}{2}$	Neither	Although they are opposites (one positive, one negative), these fractions are not reciprocals.
$\frac{3}{8}$	$\frac{3}{8}$	Parallel	Since they both have the same value for the slope, they are parallel lines.

TERMS TO KNOW

Parallel Lines

Lines that never intersect; they have identical slopes.

Perpendicular Lines

Lines that intersect at a right angle; they have opposite reciprocal slopes.

SUMMARY

The **slope of a line** is a measure of how steep it is. When reading a graph from left to right, lines with a negative slope head towards negative infinity on the y-axis and lines with a positive slope head towards positive infinity on the y-axis. The **slopes of vertical and horizontal lines** are unique. Horizontal lines have a slope of zero and vertical lines have a slope that is undefined. The **slopes of parallel and perpendicular lines** are special too. Parallel lines have the same slope and perpendicular lines have slopes that are opposite reciprocals of each other.

TERMS TO KNOW

Parallel Lines

Lines that never intersect; they have identical slopes.

Perpendicular Lines

Lines that intersect at a right angle; they have opposite reciprocal slopes.

Slope

The steepness of a line; found by dividing the change in y-coordinates by the change in x-coordinates from any two points on a line.

FORMULAS TO KNOW

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$