Isolating Variables

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WHAT'S COVERED

In this lesson, you will learn how to identify the operations needed to isolate a variable in an equation. Specifically, this lesson will cover:

1. Process of Solving an Equation

When solving an equation for a variable, our main goal is to isolate a variable. In other words, we want to get the variable by itself on one side of the equation, with all other expressions on the other side of the equals sign. In this process, we must always remember that if we perform an operation on one side of the equal sing, we must do the same on the other side of the equal sign.

EXAMPLE

$$\begin{array}{r} 3x+2=5 \\ -2 \quad -2 \end{array}$$ Solve for x by first subtracting 2 from both sides

$$3x=3$$ Divide both sides by 3

$$\frac{3x}{3}=\frac{3}{3}$$

$$x=1$$ Our Solution

BIG IDEA

Whatever we do on one side of the equation has to be done on the other side of the equation. This is known as the Rule of Equality.

2. Review of Inverse Operations

When isolating a variable, we need to keep the following in mind:

Operation	Inverse Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division

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Division	Multiplication
Powers	Roots
Roots	Powers

3. Applying Inverse Operations

A good rule of thumb is to isolate the outermost operations surrounding the variable first, working our way inwards until we isolate the variable.

↱ EXAMPLE

$20 = 2x - 8$ Solve for x by first adding 8 to both sides

$+8 \quad +8$

$28 = 2x$ Divide both sides by 2

$\frac{28}{2} = \frac{2x}{2}$

$14 = x$ Our Solution

HINT

In general, we apply the inverse operations following the reverse order of operations to isolate a variable.

4. Simplifying Before Isolating a Variable

Sometimes when we try to isolate a variable, it may be better to simplify the equation before we perform any inverse operations.

↱ EXAMPLE Suppose we want to solve for x in the following equation: $5(2x-6) = 7$.

There are two ways we can go about solving this equation. First, we can distribute 5 into the $2x$ and -6, and then isolate the variable, or we can divide both sides of the equation by 5 first, and then solve for x. Either method is valid, and you are free to use either when trying to isolate the variable. Let's take a look at how we can use both methods to solve the equation above:

By distribution:

$5(2x-6) = 7$ Solve for x by distributing 5 into $2x-6$

$10x - 30 = 7$ Add 30 to both sides

$10x = 37$ Divide both sides by 10

$x = \frac{37}{10}$ Express $\frac{37}{10}$ as a decimal

$x = 3.7$ Our Solution

Dividing by 5 first:

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$5(2x-6)=7$	Solve for x by dividing both sides by 5
$\frac{5(2x-6)}{5}=\frac{7}{5}$	5's on the left side cancel, leaving $2x-6$; express $\frac{7}{5}$ as a decimal
$2x-6=1.4$	Add 6 to both sides
$\frac{2x}{2}=\frac{7.4}{2}$	Divide both sides by 2
$x=3.7$	Our Solution

Let's look at another example where combining like terms before attempting to isolate the variable can be helpful:

↱ EXAMPLE

$2x-6=5x+3$	Solve for x by first move the x terms to one side
$-2x \quad -2x$	$5x-2x=3x$
$-6=3x+3$	Move constant terms to one side
$-3 \quad -3$	$-6-3=-9$
$-9=3x$	Divide both sides by 3
$\frac{-9}{3}=\frac{3x}{3}$	$-9\div 3=-3$
$-3=x$	Our Solution

HINT

When trying to isolate a variable, it is always a good idea to simplify the equation as much as possible before starting to isolate the variable with inverse operations. This usually means that we should combine like terms whenever possible.

SUMMARY

The **process of solving an equation** involves isolating the variable you want to solve for. When isolating a variable, it is helpful to have a **review of inverse operations**: addition and subtraction are inverse, multiplication and division are inverse and powers and roots are inverse. Keep in mind when **applying inverse operation** that this will cancel the operations around the variable. Also, in using the inverse operations, use the order of operations in reverse order. Finally, **simplifying before isolating a variable**, such as distributing or combining like-terms, can be helpful.

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